Logics for MAS: a critical overview

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IJCAI 2013, August 9, 2013

Introduction

Introduction

Multi-Agent Systems (MAS):
agents with imperfect knowledge
perform actions
in order to achieve goals

- philosophical logic/KR view:
  - what are the main concepts?
  - what properties do they have?
  - how do they relate?
- formal, logical analysis
  $\Rightarrow$ logics of action and knowledge
  $\Rightarrow$ extensions of propositional logic by *modal operators*

Introduction: modal operators of knowledge

- knowledge of individual $i \in \mathtt{Agt}$:

  $K_i\varphi$ = "agent i knows that φ"

- knowledge of group $J \subseteq \mathtt{Agt}$:

  $EK_J\varphi$ = "it is shared knowledge in J that φ"
  = "every agent in J knows that φ"

  $CK_J\varphi$ = "it is common knowledge in J that φ"
  = $EK_J\varphi \wedge EK_J EK_J\varphi \wedge EK_J EK_J EK_J\varphi \wedge \cdots$ "

  $DK_J\varphi$ = "it is distributed knowledge in J that φ"
  = "if each agent in J tells all he knows to J then $CK_J\varphi$"

Introduction: modal operators of action and ability

- nonstrategic (ceteris paribus)

$\langle \pi \rangle \varphi$ = "there is an execution of program π after which φ"

$\langle J \rangle \varphi$ = "coalition J can achieve φ (while opponents don't act)"

- strategic ('ceteris agentis','ceteris mutandis')

$\langle\langle J \rangle\rangle \varphi$ = "coalition J can achieve φ (whatever opponents do)"

$\mathrm{Stit}_J\, \varphi$ = "coalition J achieves φ (whatever opponents do)"

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Introduction: the grid of MAS logics

- aim of talk: overview the main MAS logics and highlight problematic points
  - KR point of view: which logical language?
  - semantic-free
- the grid of MAS logics:

	$S5^C$	PAL^C	$ATEL^C$
	S5	PAL	ATEL
no uncertainty		PDL, CL-PC	ATL
knowledge / action	no actions	nonstrategic	strategic

Outline

1. No uncertainty, nonstrategic actions
2. No uncertainty, strategic actions
3. Individual knowledge, no actions
4. Individual knowledge, nonstrategic actions
5. Individual knowledge, strategic actions
6. Group knowledge, no actions
7. Group knowledge, nonstrategic actions
8. Group knowledge, strategic actions

No uncertainty, nonstrategic actions: PDL

- language of Propositional Dynamic Logic PDL:
  $\langle\pi\rangle\varphi$ = "*there exists* a possible execution of π after which φ"
  $[\pi]\varphi$ = "*for every* possible execution of π..."
  where π is a program (alias complex action):

$$\pi \ ::= \ a \mid \pi;\pi \mid \pi \cup \pi \mid \pi^* \mid \varphi?$$

  $\Rightarrow$ "while φ do π" = $(\varphi?;\pi)^*;\neg\varphi?$
- in focus: reasoning about action/program effects

$$(\mathit{ActionTheory} \wedge \mathit{Init}) \rightarrow \langle a_1; \cdots ; a_n\rangle \mathit{Goal}$$

No uncertainty, nonstrategic actions: PDL

- PDL action theories must be augmented by frame axioms

$$BlockRed \rightarrow [moveBlock_{L_1,L_2}]BlockRed$$

$\Rightarrow$ PDL doesn't solve the frame problem [McCarthy & Hayes 1969]

- a lot of dedicated logical formalisms

  SitCalc, EventCalc, FluentCalc, $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{C}+$, $\mathcal{BC}$, separation logic, ...

- SitCalc basic action theories [Reiter 1991]:

$$\forall x\Big([x]BlockRed \leftrightarrow \big(x = paintRed \vee (BlockRed \wedge x \neq paintBlue)\big)\Big)$$

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DL-PA: a dialect of PDL solving the frame problem

- Reiter's basic action theories can be expressed in
  Dynamic Logic of *Propositional Assignments* DL-PA
  [van Ditmarsch, H & de Lima, JLC 2011]
  - atomic programs: assign propositional variables to formulas

$$BlockAt_{L_1} {:=} \bot$$

  - successor state axioms become DL-PA programs:

$$moveBlock_{L_1,L_2} = \big(Free?; BlockAt_{L_1} {:=} \bot; BlockAt_{L_2} {:=} \top\big)$$

  hyp.: in $\forall x \big([x]p \leftrightarrow \gamma_p(x)\big)$, if $a \notin \gamma_p(x)$ then $\gamma_p(a) \leftrightarrow p$
- nice properties [Balbiani, H & Troquard, LICS 2013]
  - complexity of satisfiability just as PDL
  - model checking as complex as satisfiability checking
  - Kleene star eliminable
  - every formula reducible to a boolean formula
- claim: DL-PA = Assembler language for logics of change...

No uncertainty, nonstrategic actions: CL-PC

- language of Coalition Logic of Propositional Control CL-PC:
  $\langle J \rangle \varphi$ = "coalition J can achieve φ by modifying its variables (while opponents don't act)"
  - each propositional variable *controlled* by some agent; action of i = change of some of i's variables (cf. bool. games)

[van der Hoek & Wooldridge, AIJ 2005; JAIR 2010]

- in focus: reasoning about nonstrategic (ceteris paribus) ability

$$(AbilityTheory \wedge Init) \rightarrow \langle \{i_1, \ldots, i_n\} \rangle Goal$$

No uncertainty, nonstrategic actions: CL-PC

- captures strategic ability
  $\langle J\rangle[\bar{J}]\varphi$ = "*J* can achieve φ *whatever the opponents in* $\bar{J}$ *do*"
- can be embedded into DL-PA:

$$\langle i\rangle\varphi = \langle\pi_{i,\varphi}\rangle\varphi$$

with $\pi_{i,\varphi}$ polynomial in φ [H et al., IJCAI 2011]

Outline

No uncertainty, strategic actions: ATL

- language of Alternating-time Temporal Logic ATL:

  $\langle\langle J\rangle\rangle\mathcal{X}\varphi$ = "the agents in J have a strategy such that *whatever the other agents do*, next φ"

  $\langle\langle J\rangle\rangle\mathcal{G}\varphi$ = "..., henceforth φ"

  $\langle\langle J\rangle\rangle\varphi\,\mathcal{U}\,\psi$ = "..., φ until ψ"

- in focus: reasoning about the existence of strategies

$$(\mathit{AbilityTheory} \wedge \mathit{Init}) \rightarrow \langle\langle\{i_1, \ldots, i_n\}\rangle\rangle \mathit{Goal}$$

ATL: the problem of strategy revocability

- problem: strategies can be canceled

  $\langle\langle i\rangle\rangle \mathcal{G}\big(married \wedge \langle\langle i\rangle\rangle \mathcal{X}\neg married\big)$ is satisfiable

  $\Rightarrow$ reason: strategies are "unsung heroes" [van Benthem]
- solution: *commit* to a strategy
  - ATL with irrevocable strategies [Ågotnes et al., TARK 2007]
  - ATL with strategy contexts [Brihaye et al., LFCS 2009]
    - make adoption and canceling of strategies explicit
    - undecidable [Troquard & Walther, JELIA 2012]
  - Strategy Logic (SL) [Mogavero et al., FSTTCS 2010]
    - uses strategy variables; undecidable
  - ATL with explicit strategies [Walther et al., TARK 2007]

    $\langle\langle\{i\}\rangle\rangle_{i:\sigma}\mathcal{G}(married \wedge \langle\langle\{i\}\rangle\rangle_{i:\sigma}\mathcal{X}\neg married) \rightarrow \bot$
- more principled: commit to an action
  - ATLEA = ATL + Explicit Actions [H, Lorini & Walther, LORI 2013]

    $\langle\langle\{i\}\rangle\rangle_{i:staymarried^\infty}\mathcal{G}(married \wedge \langle\langle\{i\}\rangle\rangle_{i:staymarried^\infty}\mathcal{X}\neg married) \rightarrow \bot$

    - same complexity as ATL

No uncertainty, strategic actions: STIT

- language of Seeing-To-It-That Logic STIT

[Belnap et al. 2001; Horty 2001]

$\text{Stit}_J\,\varphi$	=	"by following their *current strategy* the agents in J guarantee that φ is true, whatever the other agents do"
$\Diamond\varphi$	=	"it is historically possible that φ"
$\mathcal{F}\varphi$	=	"..." (temporal operators)

- in focus: reasoning about causality ('agency')

$$\textit{Cond} \rightarrow \text{Stit}_{\{i_1,\ldots,i_n\}}\,\textit{Fact}$$

- reasoning about strategic ability *à la* ATL:

  $\langle\!\langle J\rangle\!\rangle\mathcal{X}\psi = \Diamond\text{Stit}_J\,\mathcal{X}\psi$

- satisfiability undecidable [H & Schwarzentruber, AiML 2008]

Outline

Individual knowledge, no actions

- language of modal logic S5:
  $K_i\varphi$ = "agent *i* knows that φ is true"
- principles
  - $K_i\top$ (omniscience)
  - $(K_i\varphi \wedge K_i(\varphi \to \psi)) \to K_i\psi$ (omniscience)
  - $K_i\varphi \to \varphi$ (knowledge implies truth)
  - $K_i\varphi \to K_iK_i\varphi$ (positive introspection)
  - $\neg K_i\varphi \to K_i\neg K_i\varphi$ (negative introspection)
- "the" logic of knowledge?
  - generally adopted in AI
  - but. . .

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Individual knowledge, no actions

- negative introspection axiom $\neg K_i\varphi \rightarrow K_i\neg K_i\varphi$ too strong
  [Lenzen 1978, Voorbraak 1993]

  1. suppose B_iK_ip
     - i strongly believes to know p
     - should not imply K_ip
  2. suppose $\neg p$
  3. then $\neg K_ip$ (knowledge implies truth)
  4. then $K_i\neg K_ip$ (neg. introspection)
  5. then $B_i\neg K_ip$ (knowledge implies belief)
  6. $\bot$ (belief consistent)

  $\Rightarrow$ $(B_iK_ip \wedge \neg p) \rightarrow \bot$?!?

- logic of knowledge should rather be S4.2 [Lenzen 1978]
  $\Rightarrow$ dynamic epistemic logics get more involved...

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Individual knowledge, nonstrategic actions: PAL

- Public Announcement Logic PAL
  $\langle\psi!\rangle\varphi$ = "the truthful public announcement of ψ can be made and φ will be true afterwards"
- reduction axioms (aka regression):

$$\langle\psi!\rangle p \leftrightarrow \psi \wedge p$$ facts don't change (epistemic change only)

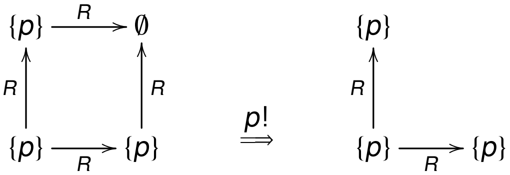
$$\langle\psi!\rangle \mathrm{K}_i\varphi \leftrightarrow \psi \wedge \mathrm{K}_i[\psi!]\varphi$$

- complexity of satisfiability:
  - same as underlying epistemic logic [Lutz, AAMAS 2006]
  - but more succinct [French et al., IJCAI 2011]

Individual knowledge, nonstrategic actions: the problem of closure under updates in PAL

- most papers choose S5 as the logic of knowledge
  - others adopt K for generality
- S5-based PAL 'works' because the set of S5 models is closed under updates by announcements
  - holds also in modal logic K
- fails in logic of belief KD45 and in logic of knowledge S4.2

  [Balbiani, van Ditmarsch & H, AiML 2012]
  - reason: confluence node may be eliminated by update



- similar problem with other modal logics

Individual knowledge, nonstrategic actions: variants of PAL

- DEL = Dynamic Epistemic Logic [Baltag & Moss, Synthese 2004]
  - agents perceive events only incompletely
    $\Rightarrow$ event models

- GAL = PAL plus Group announcements [Ågotnes et al. 2010]

  $\langle J \rangle \varphi =$ "J can achieve φ by announcing some known formulas"

  $\Rightarrow$ cf. ATL, CL

- APAL = PAL plus Arbitrary announcements

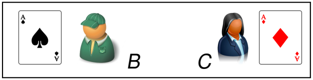
  [Balbiani et al., RSL 2008]

  $\langle ! \rangle \varphi \quad = \quad$ "there is a ψ such that $\langle \psi ! \rangle \varphi$"

Individual knowledge, nonstrategic actions: the problem of uniform choices in APAL

- You don't see *B*'s and *C*'s cards, and they only see their cards.
- Among the ace of spades and the ace of clubs, *B* has one and *C* has one, but You don't know who has which.
- You want agent *B* to know both *Spades* and *Clubs*, but not *C*.
- Is there a public announcement doing the job?

Individual knowledge, nonstrategic actions: the problem of uniform choices in APAL



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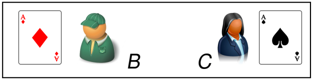


- in S5:

$$Init = K_Y Spades \wedge K_Y Clubs \wedge K_Y\big((K_B Spades \wedge \neg K_C Spades) \vee (K_B Clubs \wedge \neg K_C Clubs)\big)$$

$$Goal = K_B(Spades \wedge Clubs) \wedge \neg K_C(Spades \wedge Clubs)$$

- provable in PAL:

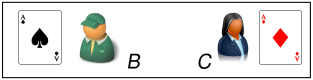
$$(K_B Spades \wedge \neg K_C Spades) \rightarrow \langle Spades {\rightarrow} Clubs! \rangle Goal$$

$$(K_B Clubs \wedge \neg K_C Clubs) \rightarrow \langle Clubs {\rightarrow} Spades! \rangle Goal$$

- so $Init \rightarrow K_Y \langle \exists ! \rangle Goal$, ...but you don't know what to say!
- in Group Announcement Logic GAL:

$K_Y \langle \{Y\} \rangle \varphi$ vs. $\langle \{Y\} \rangle K_Y \varphi$

Individual knowledge, nonstrategic actions: the problem of uniform choices in APAL



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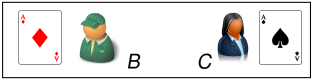

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Individual knowledge, strategic actions

- Alternating-time Temporal Epistemic Logic ATEL

  [van der Hoek & Wooldridge, Studia Logica 2003]

  $\langle\langle J\rangle\rangle\varphi$ = "coalition J can achieve φ (whatever opponents do)"

  $K_i\varphi$ = "agent $i \in \mathtt{Agt}$ knows that φ"

- problem of uniform strategies [Schobbens, ENTCS 2004]
  - same as problem of uniform choice for APAL, v.s.

$$K_i\langle\langle i\rangle\rangle \mathcal{X} safeOpen$$

- solution in ATELEA = ATEL with Explicit Actions

$$K_i\langle\langle i\rangle\rangle_{i:dial_{1234}} \mathcal{X} safeOpen$$

Individual knowledge, strategic actions

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Outline

Group knowledge, no actions

- $S5^C$ = S5 plus Common knowledge

  $CK_J\varphi$ = "it is *common* knowledge in $J \subseteq \texttt{Agt}$ that φ"

  $= EK_J\varphi \wedge EK_J EK_J\varphi \wedge EK_J EK_J EK_J\varphi \wedge \cdots$

- fixpoint axiom:

$$CK_J\varphi \leftrightarrow EK_J(\varphi \wedge CK_J\varphi)$$

- induction axiom:

$$\big(\varphi \wedge CK_J(\varphi \to EK_J\varphi)\big) \to CK_J\varphi$$

$\Rightarrow$ will be criticized in the next section

Group knowledge, no actions

- $S5^C$ = S5 plus Common knowledge

  $CK_J\varphi$ = "it is *common* knowledge in $J \subseteq \mathtt{Agt}$ that φ"

  $= EK_J\varphi \land EK_JEK_J\varphi \land EK_JEK_JEK_J\varphi \land \cdots$

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Group knowledge, no actions

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Group knowledge, nonstrategic actions

- PAL^C = PAL plus Common knowledge
- semantics: same as PAL
- accessibility relation for CK_J = greatest fixpoint of EK_J relation
  - ⇒ 'rebuilt' after each update
  - ⇒ no reduction axioms for CK_J:

$$\models CK_J[\psi!]\varphi \rightarrow [\psi!]CK_J\varphi$$
$$\not\models [\psi!]CK_J\varphi \rightarrow (\neg\psi \vee CK_J[\psi!]\varphi)$$

  - ⇒ common knowledge may 'pop up' in an unforeseeable way!

Group knowledge, nonstrategic actions: the ignorant compatriots

- Agents B and C are both Italian and don't know each other. They meet during the coffee break and start to talk in English.

$$Init = \mathrm{K}_B IT_B \wedge \mathrm{CK}_{\{B,C\}}(IT_B \rightarrow \mathrm{K}_B IT_B) \wedge (\neg IT_B \rightarrow \mathrm{K}_B \neg IT_B) \wedge \mathrm{K}_C IT_C \wedge \mathrm{CK}_{\{B,C\}}(IT_C \rightarrow \mathrm{K}_C IT_C) \wedge (\neg IT_C \rightarrow \mathrm{K}_C \neg IT_C)$$

1. first scenario:
   a third agent truthfully says: "Hey, you are both Italian!"

$$Init \rightarrow \langle IT_B \wedge IT_C ! \rangle \mathrm{CK}_{\{B,C\}}(IT_B \wedge IT_C)$$

2. second scenario:
   a third agent truthfully says: "Hey, you are compatriots!"

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[Lorini & H, 2013]

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Group knowledge, nonstrategic actions: the ignorant compatriots, ctd.

- After the announcement of $IT_B \leftrightarrow IT_C$, is it part of the common ground of the conversation that $IT_B \wedge IT_C$???
- implicit vs. explicit common knowledge

$$Init \rightarrow \langle IT_B \leftrightarrow IT_C ! \rangle \big(\mathrm{ICK}_{\{A,B\}}(IT_B \wedge IT_C) \wedge \neg \mathrm{ECK}_{\{A,B\}}(IT_B \wedge IT_C) \big)$$

- implicit common knowledge = $\mathrm{PAL}^{\mathrm{C}}$ common knowledge
  - induction axiom: OK
  - reduction axiom: KO
- explicit common knowledge: accessibility relation for ECK_J is some fixpoint, but not necessarily the greatest
  - induction axiom: KO
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$$[\psi!]\mathrm{ECK}_J\varphi \leftrightarrow (\psi \rightarrow \mathrm{ECK}_J[\psi!]\varphi)$$

Group knowledge, nonstrategic actions: the ignorant compatriots, ctd.

- After the announcement of $IT_B \leftrightarrow IT_C$, is it part of the common ground of the conversation that $IT_B \wedge IT_C$???
- implicit vs. explicit common knowledge

$$Init \to \langle IT_B \leftrightarrow IT_C ! \rangle \big(\mathrm{ICK}_{\{A,B\}}(IT_B \wedge IT_C) \wedge \neg \mathrm{ECK}_{\{A,B\}}(IT_B \wedge IT_C) \big)$$

- implicit common knowledge = $\mathrm{PAL^C}$ common knowledge
  - induction axiom: OK
  - reduction axiom: KO
- explicit common knowledge: accessibility relation for ECK_J is some fixpoint, but not necessarily the greatest
  - induction axiom: KO
  - reduction axiom: OK

$$[\psi !]\mathrm{ECK}_J \varphi \leftrightarrow (\psi \to \mathrm{ECK}_J [\psi !]\varphi)$$

Outline

Group knowledge, strategic actions

- $ATEL^C$ = ATEL plus common knowledge
- problem: which form of group knowledge required for (uniform) group strategies?
  - sometimes distributed knowledge $DK_J\varphi$
  - sometimes shared knowledge $EK_J\varphi$
  - sometimes common knowledge $CK_J\varphi$

Conclusion

	$S5^C$	PAL^C	$ATEL^C$
	S5	PAL	ATEL
no uncertainty		PDL, CL-PC	ATL
knowledge / action	no actions	nonstrategic	strategic

- revisited logics for MAS and their problems
  - S5: inadequate as a logic of knowledge
  - $S5^C$: questionable as *the* logic of common knowledge
  - APAL and ATEL: can't talk about uniform strategies
  - ATL: commitment to strategies missing

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Thanks

- the SINTELNET network (`www.sintelnet.eu`)
- based on joint work with:
  Philippe Balbiani (U. Toulouse, CNRS),
  Hans van Ditmarsch (U. Nancy, CNRS),
  Tiago de Lima (U. Artois, CNRS),
  Emiliano Lorini (U. Toulouse, CNRS),
  Frédédric Moisan (U. Toulouse),
  François Schwarzentruber (U. Rennes, ENS),
  Nicolas Troquard (CNR, Trento),
  Dirk Walther (U. Dresden)