R-Calculus:

A Logical Framework for Scientific Discovery

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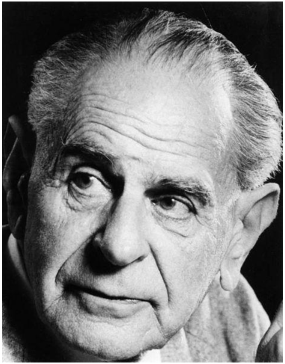
Outline

Outline



Karl Popper 1902-1994

Karl Popper published his book: The Logic of Scientific Discovery, in German in 1935 and then in English in 1959. In this book, he proposed a general process of scientific discovery.

1.1 Philosophy: General Process of Scientific Discovery

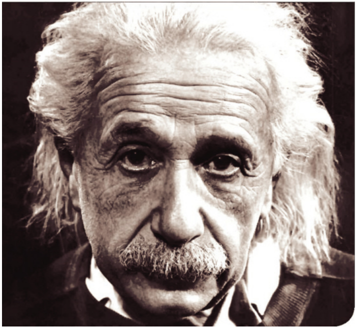
In this process:

1. A scientific theory is a consistent set of laws or principles which are conjectures extracted from and made for specifying certain domain knowledge.
2. Every such theory is subject to verification by empirical evidences (facts) which are supported by experiments and observations.
3. A theory is refuted by facts whenever its logical consequence contradicts the facts supported by experiments and observations.
4. In such case, a new scientific theory will be made by discarding the refuted laws and by proposing new conjectures.

1.2 Our interest is

- How to discover or create new theories, in particular,
- How to design mechanisms for deleting the laws or principles whose logical consequences have been refuted and
- How to extract new conjectures from experiments and observations.

1.3 A case study: Einstein's special theory of relativity



Albert Einstein, Relativity: The Special and The General Theory — A Popular Exposition, December 1916.
In the first 15 pages of this book, Einstein describes how the special theory of relativity was discovered.

Classical Physics (Γ)

During his time, Einstein had to deal with two issues. First is Classical Physics (Γ) which consists of the following laws and principles:

- GT: Galilean Transformation
- R: Principle of Relativity
- N_1, N_2, N_3: Newton's Three Laws of Motion
- E: The Law of Universal Gravitation

The logical consequence $A[c]$

According to classical physics (Galilean transformation), the velocity of a photon (propagation of light) is dependent on the velocity of motion of the body emitting the light ($A[c]$).

Fact $\neg A[c]$

Second is the fact, supported by scientific experiments and astronomical observations, that "the velocity of propagation of light does not depend on the velocity of motion of the body emitting the light." [Einstein 1916]

This means:

- the fact $\neg A[c]$ is supported by experiments and observations.

Einstein's insight

Einstein noted penetratingly that

1. The classical physics is not consistent with the fact described by $\neg A[c]$.
2. His logical intuition led him to go further to reject Galilean transformation.
3. His insight told him to accept constancy of the velocity of light as a new law.
4. The law of constancy of velocity of light led him to accept the Lorentz transformation (L) instead of Galilean's since it agrees with the law of constancy of the velocity of light.

By doing so, Einstein created his special theory of relativity:

$$\{B[c], \neg A[c], L\} \bigcup \{R,\ N_1,\ N_2, N_3,\ E\}$$

1.4 AI: Operational Process of Scientific Discovery

When an existing theory is found contradictory with facts, the discovery process starts and it consists of four steps:

1. discarding the laws which lead to contradictions,
2. forming a maximal subset of the existing theory with the remaining laws, which are consistent with facts,
3. extracting the facts supported by experiments and observations as new laws,
4. merging the new laws with the above maximal subset to establish a new scientific theory.

1.5 Operational Process of Scientific Discovery

Our Belief

It is commonly believed that scientific discovery is accomplished by great minds based on their intuitions and insight, and we believe that it can also be achieved in a large degree from logical reasoning.

Inference System

According to Bertrand Russell, for any study concerning logical analysis and reasoning, a formal inference system of logical connectives and quantifiers can be built up to conduct the logical reasoning formally.

1.6 Einstein's special theory of relativity

Formal description of Galilean transformation

"Let x represent a rigid body. If x is moving uniformly with velocity $\vec{v}$ in a straight line with respect to a co-ordinate system K and K is moving uniformly with velocity $\vec{w}$ in a straight line with respect to the second co-ordinate system K', then x is also moving uniformly in a straight line with respect to the co-ordinate system K', but its velocity is $\vec{v} + \vec{w}$. " [Einstein 1916]

$B(x)$ stands for *"x is a rigid body."*

$A(x)$ stands for *"if x is moving uniformly $\cdots$, its velocity is $\vec{v} + \vec{w}$ "*.

Galilean transformation can be represented by

$$\forall x(B(x) \to A(x)).$$

$A[c]$: formally derived logical consequence

By the modus pones rule,

$$B[c], \forall x(B(x) \to A(x)) \vdash A[c]$$

is proved.

$A[c]$ can be interpreted as: In classical physics, "the velocity of a photon (propagation of light) is dependent on the velocity of motion of the body emitting the light."

$\neg A[c]$: formal representation of the fact

- $\neg A[c]$: "The velocity of propagation of light does not depend on the velocity of motion of the body emitting the light." [Einstein 1916]
- $\neg A[c]$ is supported by experiments and observations.

The above logical inference is a formalization of Einstein's decision process about deleting Galilean transformation.

1.7 Motivation of **R**-calculus

- The motivation of designing **R**-calculus is to build a formal inference system which can be used to delete the laws in conflict with facts and to obtain maximal contractions.
- The goal is to develop a sound, complete and reachable logical framework for knowledge automation and evolution.

Outline

2.1 The Formal Language for Scientific Discovery

First-order Language $\mathcal{L}$

$\mathcal{L}$ consists of two kinds of syntactic objects: Terms and Formulas.

- Terms are defined as:

$$t \ ::= \ c \mid x \mid f(t_1, t_2, \ldots, t_n).$$

- Formulas are defined as:

$$\begin{aligned} A \ ::= \ & t_1 \doteq t_2 \mid P(t_1, t_2, \cdots, t_n) && \text{atomic formulas} \\ & \mid \neg B \mid A \wedge B \mid A \vee B \mid A \to B \mid \forall x A \mid \exists x A && \text{composite formulas} \end{aligned}$$

We choose first-order language because it is a simple, basic and fundamental formal language.

2.2 The key points of **R**-calculus

Configuration in scientific discovery

All scientists have to deal with the following two issues in their scientific discoveries:

- theories to be revised,
- facts supported by experiments and observations.

Scientific theories to be revised

In general, a scientific theory can be described by a formal theory Γ of a first-order language $\mathscr{L}$.

Facts supported by experiments

Facts supported by experiments and observations for the theory can be described also by a formal theory Δ consisting of atomic formulas and negations of atomic formulas of $\mathscr{L}$.

2.3 Configuration in scientific discovery

The components of Δ

1. Because of the rapid development of modern sensor technology, information acquired from experiments and observations may all be digitalized.
2. They are data and can be represented by constants, functions, or sets.
3. The facts extracted from the data can be represented by equations or inequalities, or predicates describing sets of data which share certain common attributes.
4. Thus, the set of facts, denoted by Δ, consists of atomic formulas and negations of atomic formulas.

2.4 **R**-configuration

Definition (R-configuration)

A configuration of each step of a scientific discovery process can be described formally by

$$\Delta \mid \Gamma$$

called an **R**-configuration.

Δ stands for the facts supported by experiments and observations.

Γ stands for the existing theory to be revised.

$\Delta \mid \Gamma$ may be read as Δ *overrides* Γ.

2.5 Inconsistent **R**-configuration

Inconsistent **R**-configuration

If Δ and Γ are inconsistent with each other, then $\Delta \mid \Gamma$ is called an inconsistent **R**-configuration and Δ is called an **R**-refutation with respect to Γ.

An inconsistent **R**-configuration is interpreted to mean that the existing theory Γ contradicts the facts Δ supported by experiments and observations, or the existing theory is refuted by facts Δ.

2.5 Inconsistent **R**-configuration

Inconsistent **R**-configuration and scientific discovery

1. Whenever an inconsistent **R**-configuration occurs, a scientific discovery arises, or the revision process starts.
2. The tasks of revision are to:
   1. delete the laws of Γ which contradict Δ,
   2. form the maximal subset Λ of Γ by the remaining laws.

2.6 **R**-contraction

Definition (R-contraction)

Let $\Delta \mid \Gamma$ be an inconsistent **R**-configuration.

If Λ is a maximal subset of Γ and is consistent with Δ, then Λ is called an **R**-contraction of Γ with respect to Δ.

- The concept of contraction originates from AGM belief revision theory.

R-contractions

Example

Let the formal theory Γ be the following set of sentences:

$$\Gamma : \{A, A \to B, B \to C\}.$$

It is not difficult to prove that $\Gamma \vdash C$. Assume that $\neg C$ is supported by experiments; then $\neg C \mid \Gamma$ is an inconsistent **R**-configuration. There are three maximal subsets of Γ that are consistent with $\neg C$:

$$\{A, A \to B\},$$
$$\{A, B \to C\},$$
$$\{A \to B, B \to C\}.$$

This shows that **R**-contraction of a formal theory with respect to its **R**-refutation is **not** unique.

2.7 **R**-transition

R-transition

The actions of deleting the laws of the existing theory can be formalized by **R**-transitions as follows:

$$\Delta \mid A, \Gamma \Longrightarrow \Delta \mid \Gamma$$

- A, Γ describes the existing theory,
- A stands for a law of the existing theory,
- $\Longrightarrow$ stands for the transformational relation.

It means that $\Delta \mid A, \Gamma$ is transformed to $\Delta \mid \Gamma$ by deleting A from the existing theory A, Γ.

2.8 Key points of **R**-calculus

- Δ in an inconsistent **R**-configuration $\Delta \mid \Gamma$ is the basis to revise the existing theory Γ.
- The goal of **R**-calculus is to delete the laws contained in Γ which lead to inconsistency with Δ.
- **R**-calculus must deduce all maximal subsets of Γ which are consistent with Δ.
- **R**-calculus is a formal inference system of logical connectives and quantifiers.

Outline

R-calculus: one axiom and six rules

R-axiom: $A, \Delta \mid \neg A, \Gamma \Longrightarrow A, \Delta \mid \Gamma$

R-$\wedge$ rule: $\dfrac{\Delta \mid A, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \wedge B, \Gamma \Longrightarrow \Delta \mid \Gamma} \qquad \dfrac{\Delta \mid B, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \wedge B, \Gamma \Longrightarrow \Delta \mid \Gamma}$

R-$\vee$ rule: $\dfrac{\Delta \mid A, \Gamma \Longrightarrow \Delta \mid \Gamma \qquad \Delta \mid B, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \vee B, \Gamma \Longrightarrow \Delta \mid \Gamma}$

R-$\rightarrow$ rule: $\dfrac{\Delta \mid \neg A, \Gamma \Longrightarrow \Delta \mid \Gamma \qquad \Delta \mid B, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \rightarrow B, \Gamma \Longrightarrow \Delta \mid \Gamma}$

R-$\forall$ rule: $\dfrac{\Delta \mid A[t/x], \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid \forall x A(x), \Gamma \Longrightarrow \Delta \mid \Gamma}$

R-$\exists$ rule: $\dfrac{\Delta \mid A[y/x], \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid \exists x A(x), \Gamma \Longrightarrow \Delta \mid \Gamma}$

R-cut rule: $\dfrac{\Gamma_1, A, \Gamma_2 \vdash C \qquad A \mapsto_{\mathcal{T}} C \qquad \Delta \mid C, \Gamma_2 \Longrightarrow \Delta \mid \Gamma_2}{\Delta \mid \Gamma_1, A, \Gamma_2 \Longrightarrow \Delta \mid \Gamma_1, \Gamma_2}$

R-calculus

R-axiom

$$A, \Delta \mid \neg A, \Gamma \Longrightarrow A, \Delta \mid \Gamma$$

- $A, \Delta \mid \neg A, \Gamma$ on the left indicates that the law $\neg A$ occurs in the existing theory $\neg A, \Gamma$.
- A on the left of $\mid$ is a fact supported by experiments and observations.
- $\Longrightarrow$ means that $\neg A$ must be deleted from the existing theory $\neg A, \Gamma$.
- The new transformed **R**-configuration is $A, \Delta \mid \Gamma$.

R-calculus

R- $\wedge$ **Rule**

$$\frac{\Delta \mid A, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \wedge B, \Gamma \Longrightarrow \Delta \mid \Gamma} \qquad \frac{\Delta \mid B, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \wedge B, \Gamma \Longrightarrow \Delta \mid \Gamma}$$

The rule on the left means that if A should be deleted from the existing theory A, Γ, then $A \wedge B$ must be deleted.

The rule may be interpreted as: if A is inconsistent with Δ, then $A \wedge B$ cannot be consistent with Δ; hence $A \wedge B$ must be deleted.

R-calculus

R-$\vee$ Rule

$$\frac{\Delta \mid A, \Gamma \Longrightarrow \Delta \mid \Gamma \qquad \Delta \mid B, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \vee B, \Gamma \Longrightarrow \Delta \mid \Gamma}$$

This rule means that if both A and B are inconsistent with Δ, then $A \vee B$ must be inconsistent with Δ. Therefore, $A \vee B$ must be deleted.

R-calculus

R- $\rightarrow$ **Rule**

$$\frac{\Delta \mid \neg A, \Gamma \Longrightarrow \Delta \mid \Gamma \qquad \Delta \mid B, \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid A \rightarrow B, \Gamma \Longrightarrow \Delta \mid \Gamma}$$

This rule means that if A is consistent with Δ, but B is not, then $A \rightarrow B$ must be inconsistent with Δ. Hence $A \rightarrow B$ should be deleted from the existing theory $A \rightarrow B, \Gamma$.

R-calculus

R- $\forall$ **Rule**

$$\frac{\Delta \mid A[t/x], \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid \forall x A(x), \Gamma \Longrightarrow \Delta \mid \Gamma}$$

$A[t/x]$ is the formula obtained from the formula $A(x)$ by substituting the free variable x with a term t. The rule means that if there exists a term t which causes $A[t/x]$ to be inconsistent with Δ, then it is impossible for $\forall x A(x)$ to be consistent with Δ. Hence $\forall x A(x)$ has to be deleted. Here the term $A[t/x]$ can be interpreted as a counter example for $\forall x A(x)$.

R-calculus

R- $\exists$ Rule

$$\frac{\Delta \mid A[y/x], \Gamma \Longrightarrow \Delta \mid \Gamma}{\Delta \mid \exists x A(x), \Gamma \Longrightarrow \Delta \mid \Gamma}$$

where y is either x or a new variable.

The rule means that if for any variable y which is either x or not present in the denominator of **R**- $\exists$ rule, and $A[y/x]$ is inconsistent with Δ, then $\exists x A(x)$ is also inconsistent with Δ and should be deleted.

R-calculus

R-cut Rule

$$\frac{\Gamma_1, A, \Gamma_2 \vdash C \qquad A \mapsto_{\mathcal{T}} C \qquad \Delta \mid C, \Gamma_2 \Longrightarrow \Delta \mid \Gamma_2}{\Delta \mid \Gamma_1, A, \Gamma_2 \Longrightarrow \Delta \mid \Gamma_1, \Gamma_2}$$

In the premises given in the *numerator* of the **R**-cut rule:

1. $\Gamma_1, A, \Gamma_2 \vdash C$ means that formula C is deduced from the existing theory Γ_1, A, Γ_2,
2. $A \mapsto_{\mathcal{T}} C$ means that A is a *necessary antecedent* of C with respect to a proof tree $\mathcal{T}$ of $\Gamma_1, A, \Gamma_2 \vdash C$,
3. $\Delta \mid C, \Gamma_2 \Longrightarrow \Delta \mid \Gamma_2$ means that C is refuted by Δ and should be deleted.

The rule means that if logical consequence C of the existing theory Γ_1, A, Γ_2 is not consistent with Δ, then A, a necessary antecedent of C, should be deleted.

R-termination

Definition

An **R**-configuration $\Delta \mid \Gamma$ is called an **R**-termination, if none of the **R**-axiom and the **R**-rules can be applied to this **R**-configuration.

Lemma

If neither **R***-axiom nor any one of the* **R***-rules can be applied to* $\Delta \mid \Gamma$*, then* Δ *is consistent with* Γ*, and vice versa.*

Outline

Verification of Einstein's theory

Inconsistent R-configuration

We use **R**-calculus to verify that the deletion of Galilean transformation is the only correct choice for establishing the special theory of relativity. Let

$$\begin{array}{rcl} \Delta & | & B[c], \forall x(B(x) \to A(x)),\ \Gamma' \\ \Gamma' & = & \{R,\ N_1,\ N_2, N_3,\ E\} \\ \Delta & = & \{B[c], \neg A[c]\}. \end{array}$$

$B[c]$: light is a photon and can be viewed as a rigid body.

$\neg A[c]$: "The velocity of propagation of light does not depend on the velocity of motion of the body emitting the light."

Δ: **R**-refutation of Γ since both $B[c]$ and $\neg A[c]$ are supported by experiments and observations and $\neg A[c]$ contradicts $A[c]$ which is a logical consequence of Γ.

Proof Tree

The proof tree of the logical verification is as following.

$$\dfrac{\dfrac{B[c], \neg A[c] \mid \neg B[c], \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma' \qquad B[c], \neg A[c] \mid A[c], \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'}{B[c], \neg A[c] \mid B[c] \to A[c], \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'}}{B[c], \neg A[c] \mid \forall x(B(x) \to A(x)), \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'}.$$

The formal logical verification

According to the **R**-axiom, the two **R**-transitions

$$\begin{array}{rcl} B[c], \neg A[c] \mid A[c], \Gamma' & \Longrightarrow & B[c], \neg A[c] \mid \Gamma' \\ B[c], \neg A[c] \mid \neg B[c], \Gamma' & \Longrightarrow & B[c], \neg A[c] \mid \Gamma' \end{array}$$

hold. We take them as the premises of **R**-$\to$ rule to obtain

$$\dfrac{B[c], \neg A[c] \mid \neg B[c], \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma' \qquad B[c], \neg A[c] \mid A[c], \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'}{B[c], \neg A[c] \mid B[c] \to A[c], \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'}.$$

This means that $B[c] \to A[c]$ must be deleted, i.e., the motion of photon does not obey Galilean transformation.

The formal logical verification

Then, by applying the **R**-$\forall$ rule to the previous **R**-transition, we have

$$\frac{B[c], \neg A[c] \mid B[c] \to A[c], \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'}{B[c], \neg A[c] \mid \forall x(B(x) \to A(x)), \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'}.$$

Hence

$$B[c], \neg A[c] \mid \forall x(B(x) \to A(x)), \Gamma' \Longrightarrow B[c], \neg A[c] \mid \Gamma'$$

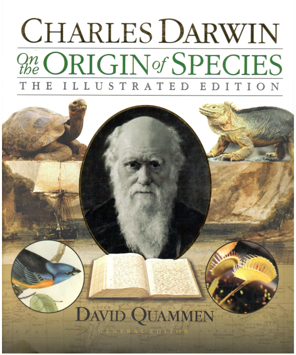
is deduced, which means that Galilean transformation is deleted.

Summary

1. It is formally verified by **R**-calculus that $\forall x(B(x) \rightarrow A(x))$ should be deleted.
2. The verification supports Einstein's logical intuition about taking out Galilean transformation.
3. The verification confirms that Einstein's decision is the only correct choice.
4. The involved calculus can be performed automatically on computer.

Outline

A formal verification of Darwin's theory



Darwin's theory of evolution

Using the **R**-cut rule, we can give a logical verification for Darwin's theory of evolution and suggest a reason why the controversies over Darwin's theory have lasted for 150 years.

Logical verification of Darwin's theory of evolution

Theory of immutability

Let us first consider the theory, described in the introduction of *The Origin of Species* [Darwin 1859], which was prevalent up to that time. We may call it the theory of immutability. We introduce the predicate symbols A, B and C to denote three basic statements given in the introduction as follows:

- A represents "*Each species has been independently created*."
- B represents "*Species are immutable*."
- C represents "*[Species] belonging to the same genera are not lineal descendants of some other and generally extinct species*."

Example of Darwin's theory of evolution

Theory of immutability

- $A \to B$ is interpreted as: "If each species has been independently created, then species are immutable."
- $B \to C$ is interpreted as: "If species are immutable, then species belonging to the same genera are not lineal descendants of some other and generally extinct species."

Example of Darwin's theory of evolution

Theory of immutability

Thus the theory of immutability given in the introduction is described by the following set of formulas:

$$\Omega : \quad \{A, A \to B, B \to C\}.$$

$$A, A \to B, B \to C \vdash C$$

can be proved by using the modus ponens rule twice.

Example of Darwin's theory of evolution

The statement of genera ancestors

Through analyzing the fossil record and by observing species that were obviously related but had diverged due to isolation in a different environment, Darwin concluded:

"*[Species] belonging to the same genera* **are** *lineal descendants of some other and generally extinct species.*" [Darwin 1859]

Let us call this **the statement of genera ancestors**. It is denoted by $\neg C$.

$\neg C$ is an **R**-refutation with respect to Ω since C is proved to be a logical consequence of Ω.

Example of Darwin's theory of evolution

The principle of natural selection

Then Darwin turned his attention to

"*how the innumerable species inhabiting this world have been modified*." [Darwin 1859]

For a long time, he studied how species in a genera had adapted to different environments. Based on his observation, he proposed the principle of natural selection in his book [Darwin 1859]. It is expressed as follows:

Example of Darwin's theory of evolution

The principle of natural selection

"*As many more individuals of each species are born than can possibly survive; and as, consequently, there is a frequently recurring struggle for existence, it follows that any being, if it vary however slightly in any manner profitable to itself, under the complex and sometimes varying conditions of life, will have a better chance of surviving, and thus be naturally selected*." [Darwin 1859].

Example of Darwin's theory of evolution

The principle of natural selection

Let us introduce the following predicate symbols:

- E to denote the sentence "*many more individuals of each species are born than can possibly survive, $\cdots$ for existence*".
- F to denote the sentence "*any being, $\cdots$ will have a better chance of surviving*".

The principle of natural selection can be described by the formula

$$E \to F.$$

Example of Darwin's theory of evolution

Darwin's main contribution

The statement of genera ancestors and the principle of natural selection which are described by

$$\{\neg C, E \to F\}.$$

Example of Darwin's theory of evolution

Darwin's theory of evolution

In the introduction of *The Origin of Species* [Darwin 1859], Darwin claimed that

"I can entertain no doubt, after the most deliberate study and dispassionate judgment of which I am capable, that the view which most naturalists entertain, and which I formerly entertained namely, that each species has been independently created is erroneous."

In other words, Darwin believes that A is erroneous and $\neg A$ is correct.

Example of Darwin's theory of evolution

Darwin's theory of evolution

Then Darwin built his theory of evolution as follows:

- take his main contribution $\{\neg C, E \to F\}$ as a base,
- delete A from the theory of immutability as he entertained,
- merge his main contribution with the remaining part of the theory of immutability $\{A \to B, B \to C\}$, and finally,
- build his theory of evolution

$$\Sigma_1 = \{A \to B, B \to C, \neg C, E \to F\}.$$

Let us show how to verify the process of discovery of Darwin's theory of evolution by **R**-calculus.

Example of Darwin's theory of evolution

Let Δ be $\{\neg C\}$.

Let Γ be $\{A, A \to B, B \to C\}$.

$\Delta \mid \Gamma$ is an inconsistent **R**-configuration. It can be transformed by applying the **R**-cut rule as follows: Let

$$\Gamma_1 = \emptyset, \quad \Gamma_2 = \{A \to B, B \to C\}.$$

Then all the three premises of the **R**-cut rule hold because

1. $\Gamma_1, A, \Gamma_2 \vdash C$;
2. A is a necessary antecedent of C, i.e. $A \mapsto C$ holds;
3. $\neg C \mid C, \Gamma_2 \Longrightarrow \neg C \mid \Gamma_2$ by **R**-axiom.

Thus, by applying the **R**-cut rule,

$$\neg C \mid \Gamma_1, A, \Gamma_2 \Longrightarrow \neg C \mid \Gamma_1, \Gamma_2$$

holds.

Example of Darwin's theory of evolution

Darwin's theory of evolution

This means that A should be deleted from the theory of immutability. Let

$$\Lambda_1 \quad \text{be} \quad \{A \to B, B \to C\}.$$

Λ_1 is the union of Γ_1, Γ_2, and is an **R**-contraction of Γ with respect to $\neg C$. In other words, Λ_1 is a maximal subset of Γ which is consistent with $\neg C$.

Since neither B nor C is a logical consequence of Λ_1, we can merge Λ_1 with Darwin's main contribution $\{\neg C, E \to F\}$ and obtain

$$\Sigma_1 : \ \{\neg C\} \cup \{A \to B, B \to C\} \cup \{E \to F\},$$

which is Darwin's theory of evolution.

Example of Darwin's theory of evolution

Darwin's theory of evolution

Applying the rule of converse-negation, we have

$$(B \to C) \equiv (\neg C \to \neg B),$$
$$(A \to B) \equiv (\neg B \to \neg A).$$

Thus

$$\Sigma_1 = \{\neg C, \neg C \to \neg B, \neg B \to \neg A, E \to F\}.$$

By applying the modus ponens rule twice, we obtain

$$\Sigma_1 \vdash \neg A,$$

i.e., $\neg A$ is a logical consequence of Σ_1.

This completes a formal verification of Darwin's entertain: "each species has been independently created is erroneous".

Example of Darwin's theory of evolution

1. Σ_1 contains Darwin's main contribution $\{\neg C, E \to F\}$.
2. Σ_1 inherits the principles of the theory of immutability of species $A \to B$ and $B \to C$.
3. verifies: "formerly entertained namely, that each species has been independently created is erroneous".
4. Σ_1 is a solution deduced formally by **R**-calculus.
5. This indicates that the above symbolic deduction is a logical verification of Darwin's theory of evolution (Σ_1).

Example of Darwin's theory of evolution

The reason of controversy Σ_2

In fact, we can derive two other logically rational **R**-contractions of Γ with respect to Δ [Li]:

$$\begin{aligned} \Lambda_2: &\quad \{A, B \to C\}, \\ \Lambda_3: &\quad \{A, A \to B\}. \end{aligned}$$

Putting $\neg C$ and $E \to F$ together with Λ_2 and Λ_3, we obtain two other theories of evolution:

$$\begin{aligned} \Sigma_2: &\quad \{\neg C\} \cup \{A, B \to C\} \cup \{E \to F\}, \\ \Sigma_3: &\quad \{\neg C\} \cup \{A, A \to B\} \cup \{E \to F\}. \end{aligned}$$

Example of Darwin's theory of evolution

The reason of controversy Σ_2

Let us consider Σ_2.

1. Σ_2 contains $\neg C$ and $E \rightarrow F$, i.e., Σ_2 includes Darwin's two main contributions: the statement of genera ancestors and the principle of natural selection. It is another version of the theory of evolution.
2. Σ_2 contains A. This means that the theory Σ_2 believes that "*each species has been independently created*" is correct.

The existence of Σ_2 provides a logical reason why the controversies over Darwin's theory have lasted for 150 years.

Outline

1. Motivation
2. Key points of **R**-calculus
3. **R**-calculus
4. Formal verification of Einstein's special theory of relativity
5. Formal verification of Darwin's theory of evolution
6. Conclusion

Reachability

Definition (R-reachability)

If for any given inconsistent **R**-configuration $\Delta \mid \Gamma$ and an arbitrary **R**-contraction Γ' of Γ with respect to Δ, there always exists an **R**-transition sequence such that

$$\Delta \mid \Gamma \Longrightarrow^* \Delta \mid \Gamma'$$

is provable, and $\Delta \mid \Gamma'$ is an **R**-termination, then we say that **R**-calculus is **R**-reachable.

Theorem (Reachability)

***R**-calculus is **R**-reachable.*

Soundness

Definition (R-soundness)

Let $\Delta \mid \Gamma$ be an inconsistent **R**-configuration and Γ' be an **R**-contraction of Γ with respect to Δ. If there exists a provable **R**-transition sequence

$$\Delta \mid \Gamma \Longrightarrow^{*} \Delta \mid \Gamma',$$

then there exists a model **M** of **R**-refutation such that both $\mathbf{M} \models \Delta$ and $\Gamma_{\mathbf{M}(\Delta)} = \Gamma'$ hold.

Theorem (Soundness)

R-*calculus is* **R**-*sound.*

Completeness

Definition (R-completeness)

If for an arbitrary inconsistent **R**-configuration $\Delta \mid \Gamma$ and an arbitrary model **M** of **R**-refutation of Γ with respect to Δ, there always exists a provable **R**-transition sequence

$$\Delta \mid \Gamma \Longrightarrow^* \Delta \mid \Gamma_{\mathbf{M}(\Delta)},$$

then we say that **R**-calculus is **R**-complete.

Theorem (Completeness)

R-*calculus is* **R**-*complete.*

Basic Theorem of Testing

Theorem

Let Δ be an arbitrary formal theory consisting of finitely many atomic formulas or negations of atomic formulas, and Γ be a finite set of arbitrary formulas. If Γ' is an arbitrary maximal subset of Γ that is consistent with Δ, then there exists an **R***-transition sequence*

$$\Delta \mid \Gamma \Longrightarrow^* \Delta \mid \Gamma'$$

that is provable.

Thanks

Thank You!

Reference

All the proofs can be found in the following book.

Li, Wei, *Mathematical Logic: Foundations for Information Science.* Birkhäuser Publisher, 2010.

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R′-rules

R′-axiom

$$\Delta \mid A, \neg A, \Gamma \Longrightarrow \Delta \mid A, \Gamma \qquad \Delta \mid A, \neg A, \Gamma \Longrightarrow \Delta \mid \neg A, \Gamma$$

where A is an atomic formula, $A \notin \Delta$ and $\neg A \notin \Delta$.

R′-$\wedge$ rule

$$\frac{\Delta \mid C, A, B, \Gamma \Longrightarrow \Delta \mid A, B, \Gamma}{\Delta \mid C, A \wedge B, \Gamma \Longrightarrow \Delta \mid A \wedge B, \Gamma}$$

R′-rules

R′-∨ rule

$$\frac{\Delta \mid C, A, \Gamma \Longrightarrow \Delta \mid A, \Gamma \qquad \Delta \mid C, B, \Gamma \Longrightarrow \Delta \mid C, \Gamma}{\Delta \mid C, A \vee B, \Gamma \Longrightarrow \Delta \mid A \vee B, \Gamma}$$

$$\frac{\Delta \mid C, A, \Gamma \Longrightarrow \Delta \mid C, \Gamma \qquad \Delta \mid C, B, \Gamma \Longrightarrow \Delta \mid B, \Gamma}{\Delta \mid C, A \vee B, \Gamma \Longrightarrow \Delta \mid A \vee B, \Gamma}$$

R′-→ rule

$$\frac{\Delta \mid C, \neg A, \Gamma \Longrightarrow \Delta \mid \neg A, \Gamma \qquad \Delta \mid C, B, \Gamma \Longrightarrow \Delta \mid C, \Gamma}{\Delta \mid C, A \rightarrow B, \Gamma \Longrightarrow \Delta \mid A \rightarrow B, \Gamma}$$

$$\frac{\Delta \mid C, \neg A, \Gamma \Longrightarrow \Delta \mid C, \Gamma \qquad \Delta \mid C, B, \Gamma \Longrightarrow \Delta \mid B, \Gamma}{\Delta \mid C, A \rightarrow B, \Gamma \Longrightarrow \Delta \mid A \rightarrow B, \Gamma}$$

$\mathbf{R}'$-rules

$\mathbf{R}'$-$\neg$ rule

$$\frac{\Delta \mid C, A', \Gamma \Longrightarrow \Delta \mid A', \Gamma}{\Delta \mid C, A, \Gamma \Longrightarrow \Delta \mid A, \Gamma}$$

where A and A' are specified by the following table:

A	$\neg(D \wedge E)$	$\neg(D \vee E)$	$\neg\neg D$	$\neg(D \to E)$	$\neg\forall x D(x)$	$\neg\exists x D(x)$
A'	$\neg D \vee \neg E$	$\neg D \wedge \neg E$	D	$D \wedge \neg E$	$\exists x \neg D(x)$	$\forall x \neg D(x)$

R′-rules

R′-∀ rule

$$\frac{\Delta \mid C, A[t/x], \forall x A(x), \Gamma \Longrightarrow \Delta \mid A[t/x], \forall x A(x), \Gamma}{\Delta \mid C, \forall x A(x), \Gamma \Longrightarrow \Delta \mid \forall x A(x), \Gamma}$$

where t is a term.

R′-∃ rule

$$\frac{\Delta \mid C, A[y/x], \Gamma \Longrightarrow \Delta \mid A[y/x], \Gamma}{\Delta \mid C, \exists x A(x), \Gamma \Longrightarrow \Delta \mid \exists x A(x), \Gamma}$$

where y is either x or an eigen-variable, that is, the variable y is different from all the variables in the denominator of the rule.