Belief Revision
From 1985 to 2013

Eduardo Fermé
The Problem
Belief Revision: Two examples

(Gärdenfors 1988)

Beliefs:

- I gave my wife a diamond ring
- Diamonds scratch glasses
- My window is made of glass

Actions:

- I gave my wife a diamond ring
- My wife tries to scratch the windows with the ring
- Nothing happens
Belief Revision: Two examples

(Gärdenfors & Rott 1995)

Beliefs:
- The bird caught in the trap is a swan
- The bird caught in the trap comes from Sweden
- Sweden is part of Europe
- All European swans are white

Consequences:
- The bird caught in the trap is white

New information:
- The bird caught in the trap is black

Which sentence(s) would you give up?
Belief Revision: Two examples

Some Conclusions:

- Consistency
- Minimal Change
- Logic is not enough to make a decision
Problem arises in several areas …

**Databases:** New entry can be inconsistent with the database. Minimal ways to erase data.

**Robotics:** Sensor information can be inconsistent with plans.

**Diagnosis:** Device behaviour inconsistent with device description
Problem arises in several areas …

**Argumentation:** The construction of argument may implies changes of beliefs in agents

**Ontologies:** an ontology will be expected to evolve, either as domain information is corrected and refined, or in response to a change in the underlying domain.
With the help of my friends Raul Carnota and Ricardo Rodríguez.
(AGM Theory and Artificial Intelligence. 2011)
The Beginning

Philosophy
Philosophy

1977, “Rational Conceptual Change”

1977, “Subjunctives, Dispositions and Chances”

1971, “Explanation and understanding”
Philosophy

Deontic Logic

Philosophy of law

Epistemology

Cognitive Science

Modal Logic

Deontic Logic
Philosophy: Alchourrón and Makinson

– To analyse the logical structure of the derogation procedure of a norm contained in a legal code.
– To find the general principles that any derogation should satisfy.
– To define a family of all the possible derogations.

"Hierarchies of Regulations and their Logic" 1981
Philosophy: Alchourrón and Makinson

– Given a code A, create a partial order between the norms of A and induce an order on the set of parts of A. The maximal sets of A that did not involve the standard were called "remainders".

“Hierarchies of Regulations and their Logic” 1981
Philosophy: Alchourrón and Makinson

– The problem was not limited only to a set of norms.

• The set A might be an arbitrary set of formulae and the problem now was how to eliminate one of the formulae or one of the consequence of the set.

“On the Logic of Theory Change: Contraction functions and their associated Revision functions”, 1982
Philosophy: Alchourrón and Makinson

– In the paper two different ways to contract a theory by means of remainder sets was analyzed.
– Maxichoice and Full Meet

“On the Logic of Theory Change: Contraction functions and their associated Revision functions”, 1982
Philosophy: Gärdenfors

- He was looking for a model for *Explanations*
- Gärdenfors thought that *Explanations* can be expressed as different types of conditional sentences
- Gärdenfors receives an important influence from the philosophers Levi and Harper, leading him to make a thorough study of epistemic conditionals.

“Conditional and Change of Belief”, 1978
“A pragmatic approach to explanations”, 1980
Philosophy: Gärdenfors

– He looking for a semantic for the epistemic conditionals.
– This semantic must be based on belief states and belief changes.

“An epistemic approach to conditionals”, 1981
“Rules for rational changes of beliefs” 1982
The Beginning

Belief Revision in Artificial Intelligence
The AI crisis in the ´80

• Allen Newell pointed out three indicators:

“A first indicator comes from our continually giving to representation a somewhat magical role.

What is indicative of underlying difficulties is our inclination to treat representation like a homunculus, as the locus of real intelligence”.

“The Knowledge Level” 1981
The AI crisis in the ’80

“A second indicator is the great theorem-proving controversy of the late sixties and early seventies. Everyone in AI has some knowledge of it, no doubt, for its residue is still very much with us. It needs only brief recounting.”

“The Knowledge Level” 1981
The AI crisis in the ’80

- The results of a questionnaire promoted in 79/80 by Brachman & Smith which was sent to the AI community

“The main result was overwhelming diversity - a veritable jungle of opinions. There is no consensus on any question of substance.”

“As one [of the respondents] said, “Standard practice of representation of knowledge is the scandal of AI.”

“The Knowledge Level” 1981
The AI crisis in the ’80

– Knowledge Representation and Reasoning (KRR) must be a priority in the AI agenda.
– He postulates the existency of a “Knowledge Level”

“...there exists a distinct computer system level, lying immediately above the symbol level, which is characterized by knowledge as the medium and the principle of rationality as the law of behavior.”

“The Knowledge Level” 1981
KRR

– Newell's work had an enormous influence on AI researchers: Brachman, Levesque, Moore, Halpern, Moses, Lifschitz, Vardi, Fagin, Ullman, Shapiro, Borgida, Winslett, etc.

“The ability of the database user to modify the content of the database, the so-called update operation, is fundamental to all database management systems”.

“First we consider the problem of updating arbitrary theories by inserting into them or deleting from them arbitrary sentences”

“when replacing an old theory by a new one we wish to minimize the change in the theory”
TARK (Theoretical Aspects of Rationality and Knowledge)

• Originally planned as a little workshop.
• Attended 40 researchers and other 250 integrate the email list.

“…included computer scientists, mathematicians, philosophers and linguists”
[…]
“given the evident interest in the area by groups so diverse, it seemed appropriate a conference, particularly one that could increase the knowledge of the workers of one field about the work developed in other fields”

Moshe Vardi
Looking for a model in AI

• Reasoning About Knowledge: An Overview (Halpern). TARK 86 (March 19-22 California) Keynote

"Most of the work discussed above has implicitly or explicitly assumed that the messages received are consistent. The situation gets much more complicated if messages may be inconsistent."

“This quickly leads into a whole complex of issues involving belief revision and reasoning in the presence of inconsistency. Although I won't attempt to open this can of worms here, these are issues that must eventually be considered in designing a knowledge base"
• In 1988, Gärdenfors e Makinson presented the AGM model on TARK 88

• The can was opened …
… let’s go to see inside.
The AGM Model
AMG Model

**Belief Set:** Set of sentences closed under logical consequence $Cn$.

$Cn$ satisfies:

- inclusion ($X \subseteq Cn(X)$)
- idempotence ($Cn(Cn(X)) = Cn(X)$)
- monotony ($Cn(X) \subseteq Cn(Y)$ if $X \subseteq Y$)

as well as supraclassicality, deduction and compactness.

Consequently for every theory $K$ we have that: $Cn(K) = K$. 
AMG Model

**Belief Base:** Set of sentences.

**Finite Belief Base:** Finite set of sentences. We identify the base $K$ with a formula $\varphi$ which is the conjunction of the formulae of $K$.

Three basic epistemic attitudes are assumed:

If $K \vdash \alpha$, $\alpha$ is accepted.

If $K \vdash \neg \alpha$, $\alpha$ is rejected.

If neither $K \vdash \alpha$ nor $K \vdash \neg \alpha$, $\alpha$ is undetermined.
The three basic operations of the AGM model, that correspond with an attitude towards the input sentence $\alpha$, are the following:

**Expansion:** This operation is in charge of incorporating sentences in the original set, without eliminating any sentence from it. It allows the passage from an epistemic state in which a belief is undetermined to another epistemic state in which the belief is accepted or rejected.

**Contraction:** This operation eliminates sentences from the original set without incorporating any new ones. It allows the passage from an epistemic state in which a belief is accepted or rejected to another epistemic state in which the belief is undetermined.

**Revision:** This operation incorporates a sentence in the original set, but it can eliminate some beliefs in order to preserve consistency of the revised set. It allows the passage from an epistemic state in which a belief is accepted (rejected) to another state in which the belief is rejected (accepted).
AGM

5 different equivalent presentations

- Postulates
- Partial Meet
- Safe/Kernel
- Possible Worlds
- Epistemic Entrenchment
AGM

5 different equivalent presentations

Postulates

Partial Meet

Safe/Kernel

Possible Worlds

Epistemic Entrenchment
Expansion

\[ K + \alpha = C_n(K \cup \{\alpha\}) \]
Postulates for Contraction

**Closure** \( K - \alpha \) is a belief set whenever \( K \) is a belief set.

**Success** If \( \nvdash \alpha \), then \( K - \alpha \nvdash \alpha \).

**Inclusion** \( K - \alpha \subseteq K \).

**Vacuity** If \( K \nvdash \alpha \), then \( K \subseteq K - \alpha \).

**Extensionality** If \( \vdash \alpha \leftrightarrow \beta \) then \( K - \alpha = K - \beta \).

**Recovery** \( K \subseteq (K - \alpha) + \alpha \).

**Conjunctive factoring** \( K - (\alpha \land \beta) = \begin{cases} K - \alpha, \text{ or} \\ K - \beta, \text{ or} \\ K - \alpha \cap K - \beta \end{cases} \)
Postulates for Revision

Closure: $K \star \alpha$ is a belief set whenever $K$ is a belief set.
Success: $\alpha \in K \star \alpha$.
Inclusion: $K \star \alpha \subseteq K + \alpha$.
Vacuity: If $K \not\vdash \neg \alpha$ then $K \star \alpha = K + \alpha$.
Consistency: $\not\vdash \neg \alpha$ then $K \star \alpha \neq \bot$.
Extensionality: If $\vdash \alpha \iff \beta$ then $K \star \alpha = K \star \beta$.

Disjunctive factoring $K \star (\alpha \lor \beta) = \begin{cases} 
K \star \alpha, & \text{or} \\
K \star \beta, & \text{or} \\
K \star \alpha \cap K \star \beta 
\end{cases}$
Contraction vs. Revision

Levi’s Identity: $K*\alpha = (K\neg\alpha)+\alpha$.

Harper’s Identity: $K\neg\alpha = K \cap K*\neg\alpha$. 
Assuming finite representability, $K$ must be expressed by a single sentence $\varphi$. Katsuno and Mendelzon reformulate the AGM postulates for such case.

(R1) $\varphi \circ \alpha \vdash \alpha$

(R2) If $\varphi \land \alpha \nvdash \bot$ then $\varphi \circ \alpha \equiv \varphi \land \alpha$

(R3) If $\alpha \nvdash \bot$ then $\varphi \circ \alpha \nvdash \bot$

(R4) If $\varphi_1 \equiv \varphi_2$ and $\alpha_1 \equiv \alpha_2$ then $\varphi_1 \circ \alpha_1 \equiv \varphi_2 \circ \alpha_2$

(R5) $(\varphi \circ \alpha) \land \psi \vdash \varphi \circ (\alpha \land \psi)$

(R6) If $(\varphi \circ \alpha) \land \psi \nvdash \bot$ then $\varphi \circ (\alpha \land \psi) \vdash (\varphi \circ \alpha) \land \psi$
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Partial Meet Contraction

Based on the inclusion-maximal subsets of $K$ that do not imply $\alpha$.

$K \perp \alpha$ (K remainder $\alpha$)

$H \in K \perp \alpha$ if and only if:

\[
\begin{cases}
H \subseteq K \\
H \nvdash \alpha
\end{cases}
\]

There is no set $H'$ such that $H \subseteq H' \subseteq K$ and $H' \nvdash \alpha$
Partial Meet Contraction: Special Cases

There are two limiting cases: 1) By selecting just one element (Maxichoice) and 2) By choosing what all the members of the set have in common (Full meet contraction).

If \( \alpha \) is a Maxichoice, then it satisfies

**Saturability** If \( \alpha \in K \), then for any \( \beta \in L \), either \( \alpha \lor \beta \in K-\alpha \) or \( \alpha \lor \neg \beta \in K-\alpha \)

If \( \alpha \) is a full meet contraction then

\[ K-\alpha = K \cap Cn(\neg \alpha) \]
Partial Meet Contraction

\( \gamma \) is a selection function for \( K \) if and only if:

- If \( K \perp \alpha \neq \emptyset \), then \( \emptyset \in \gamma(K \perp \alpha) \subseteq K \perp \alpha \).
- If \( K \perp \alpha = \emptyset \), then \( \gamma(K \perp \alpha) = K \).

\( \gamma \) is relational if and only if there is a relation \( \sqsubseteq \) such that for all sentences \( \alpha \), if \( K \perp \alpha \) is non-empty, then

\[
\gamma(K \perp \alpha) = \{ B \in K \perp \alpha \mid C \sqsubseteq B \text{ for all } C \in K \perp \alpha \}
\]

\( \gamma \) is transitively relational if and only if this hold for some transitive relation \( \sqsubseteq \).
Partial Meet Functions

\[ K \sim_\gamma \alpha = \cap_\gamma (K \perp \alpha) \]

\[ K \ast \alpha = (K \sim_\gamma \neg \alpha) + \alpha \]
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Kernel / Safe Contraction

Based on the inclusion-minimal subsets of $K$ that imply $\alpha$

$K \sqsubseteq \alpha$ (kernel set of $K$ with respect to $\alpha$)

$A \in K \sqsubseteq \alpha$ if and only if:

\[
\begin{cases} 
A \subseteq K \\
A \vdash \alpha \\
\text{If } B \subseteq A \text{ then } B \nvdash \alpha
\end{cases}
\]
An incision function $\sigma$ for $K$ is a function such that for all sentences $\alpha$:

\[
\begin{align*}
\sigma(K \not\models \alpha) &\subseteq \bigcup(K \not\models \alpha) \\
\text{If } \emptyset \neq A \in K \not\models \alpha, \text{ then } A \cap \sigma(K \not\models \alpha) \neq \emptyset
\end{align*}
\]
Kernel Contraction

\[ K_{-\sigma\alpha} = Cn(K \setminus \sigma(K \perp \alpha)) \]

\[ K\star\alpha = Cn(K \setminus \sigma(K \perp -\alpha)) + \alpha \]
Safe Contraction

$K$ is ordered according to a relation $\prec$

$\beta \prec \delta$ means that $\delta$ should be retained rather than $\beta$ if we have to give up one of them.

$\prec$ must be an acyclic, irreflexive and asymmetric relation.

$\prec$ is virtually connected if and only if for all $\alpha, \beta, \delta \in K$: if $\alpha \prec \beta$ then either $\alpha \prec \delta$ or $\delta \prec \beta$.

$\prec$ is regular if and only if it satisfies:
- continuing-up (If $\alpha \prec \beta$ and $\beta \vdash \delta$, then $\alpha \prec \delta$).
- continuing-down (If $\alpha \vdash \beta$ and $\beta \prec \delta$, then $\alpha \prec \delta$).
Safe Contraction

Any sentence $\beta$ in a belief set $K$ is safe with respect to $\alpha$ if and only if $\beta$ is not minimal under $\preceq$ with respect to the elements of any $A \in K \parallel \alpha$. The set of all safe sentences of $K$ respect to $\alpha$ is denoted by $K/\alpha$.

**Definition**

$K \sim \alpha$ is a safe contraction, based on a regular and virtually connected hierarchy $\preceq$, if and only if:

$$K \sim \alpha = Cn(K/\alpha)$$
AGM

5 different equivalent presentations

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Epistemic Entrenchment
Epistemic Entrenchment

*Epistemic entrenchment* is a binary relation $\leq$ on the sentences in the belief set $K$ such that in contraction, giving up beliefs with lower entrenchment is preferred to giving up those with higher entrenchment.
(EE1) **Transitivity** If $\alpha \leq_K \beta$ and $\beta \leq_K \delta$, then $\alpha \leq_K \delta$.

(EE2) **Dominance** If $\alpha \vdash \beta$, then $\alpha \leq_K \beta$.

(EE3) **Conjunctiveness** $\alpha \leq_K (\alpha \wedge \beta)$ or $\beta \leq_K (\alpha \wedge \beta)$.

(EE4) **Minimality** If $K \not\vdash \bot$, then $\alpha \notin K$ if and only if $\alpha \leq_K \beta$ for all $\beta$.

(EE5) **Maximality** If $\beta \leq_K \alpha$ for all $\beta$, then $\vdash \alpha$. 
Epistemic Entrenchment

\[(C \leq) \quad \alpha \leq_K \beta \text{ if and only if } \alpha \notin K-(\alpha \land \beta) \text{ or } \vdash (\alpha \land \beta).\]

\[(-G) \quad \beta \in K-\alpha \text{ if and only if } \beta \in K \text{ and either } \vdash \alpha \text{ or } \alpha <_K (\alpha \lor \beta).\]

\[(C \leq^*) \quad \alpha \leq_K \beta \text{ if and only if: If } \alpha \in K^*-(\alpha \land \beta) \text{ then } \beta \in K^*-(\alpha \land \beta).\]

\[(*_{EBR}) \quad \beta \in K^*\alpha \text{ if and only if either } (\alpha \rightarrow \neg \beta) <_K (\alpha \rightarrow \beta) \text{ or } \alpha \vdash \bot.\]
AGM

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5 different equivalent presentations

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Possible Worlds

\[\|K\| = \{X \mid K \subseteq X \in \mathcal{L} \downarrow \uparrow \}\]
\[\|\alpha\| = \|Cn(\{\alpha\})\|\]

\[\|K + \alpha\| = \|K\| \cap \|\alpha\|\]
\[\|K - \alpha\| = \text{some superset of } \|K\| \text{ that includes at least one } \neg\alpha\text{-world.}\]
\[\emptyset \subseteq \|K \ast \alpha\| \subseteq \|\alpha\|\]
Possible Worlds

Definition

Let $X$ be a proposition. A propositional selection function for $X$ is a function $f$ such that for all sentences $\alpha$:

1. $f(\|\alpha\|) \subseteq \|\alpha\|$
2. If $\|\alpha\| \neq \emptyset$ then $f(\|\alpha\|) \neq \emptyset$.
3. If $X \cap \|\alpha\| \neq \emptyset$, then $f(\|\alpha\|) = X \cap \|\alpha\|$.

$\|K - \alpha\| = \|K\| \cup f(\|\lnot \alpha\|)$

$\|K \ast \alpha\| = f(\|\alpha\|)$
Possible Worlds

The graphics style used is due to Sebastien Konieczny. Thanks Sebastien!
Possible Worlds

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Spheres System

Definition

$\$ is a system of spheres if and only if it satisfies:

$1 \emptyset \neq \$ \subseteq \mathcal{P}(\mathcal{L} \perp \perp)$,
$2 \cap \$ \in \$,$
$3 \text{If } G, G' \in \$, then $G \subseteq G'$ or $G' \subseteq G$,
$4 \cup \$ \in \$,$
$5 \text{If } \|\alpha\| \cap (\cup \$) \neq \emptyset, \text{ then } S_\alpha \in \$ \text{ and } S_\alpha \cap \|\alpha\| \neq \emptyset, \text{ and}$
$6 \mathcal{L} \perp \perp \in \$,

where $S_\alpha = \cap \{G \in \$ \mid G \cap \|\alpha\| \neq \emptyset\}$.

$f$ is sphere-based if and only if for all $\alpha$:

If $\|\alpha\| \neq \emptyset$, then $f(\|\alpha\|) = S_\alpha \cap \|\alpha\|$.
Faithful Assignment

Alternatively a sphere system can be characterized by a total preorder $\leq$ between worlds, i.e. a reflexive, transitive and total relation on $\mathcal{W}$.

A faithful assignment is a function mapping each base $\varphi$ to a pre-order $\leq_\varphi$ such that:

1. If $\omega \models \varphi$ and $\omega' \models \varphi$, then $\omega \preceq_\varphi \omega'$
2. If $\omega \models \varphi$ and $\omega' \not\models \varphi$, then $\omega <_\varphi \omega'$
3. If $\varphi \equiv \varphi'$, then $\leq_\varphi = \leq_{\varphi'}$
Faithful Assignment

An operator $\ast$ is a revision operator that satisfies (R1)-(R6) if and only if there exists a faithful assignment that maps each base $\varphi$ to a total pre-order $\leq_\varphi$ such that

$$\text{mod } \varphi \ast \alpha = \min(\text{mod}(\alpha), \leq_\varphi)$$
Spheres System / Faithful Assignment

Grove’s Notation

Konieczny’s Notation
Spheres System / Faithful Assignment

Grove’s Notation

NMR Notation
Spheres System / Faithful Assignment

Grove’s Notation

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Grove’s Notation

NMR Notation
Spheres System / Faithful Assignment

Grove’s Notation

NMR Notation
28 years of AGM
AGM
28 years

- Foundation and Criticisms
- Computability and Implementations
- Applications and Connections
- Alternative Operators
- Extensions
AGM
28 years

Foundation and Criticisms

- Foundations
- Recovery
- Success
- The use of Belief Sets
- Lack of Information in the belief sets
- Others

Extensions

Computability and Implementations

Alternative Operators

Applications and Connections
AGM
28 years

- Belief Bases
- Iteration
- Multiple Change
- Probability and possibility
- Ranking
- Extension on the language
- Others

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  - Change in the strength of belief
  - Abductive Models
  - Merging
  - Others

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AGM 28 years

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Computability and Implementations
- Non-Monotonic Logic
- Defeasible Logic
- Modal Logic
- Game Theory
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- Others

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AGM 28 years

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CRITICISMS

RECOVERY
**Minimal Change**

**Fullness** If $\beta \in K$ and $\beta \notin K-\alpha$ then $\not\models \alpha$ and $\beta \rightarrow \alpha \in K-\alpha$

**Saturability** If $\alpha \in K$, then for any $\beta \in \mathcal{L}$, either $\alpha \lor \beta \in K-\alpha$ or $\alpha \lor \lnot \beta \in K-\alpha$

**Example:** I believe that “it is four o’clock” ($\alpha$). Then I also believe that “it is four o’clock or life exists after death”($\alpha \lor \beta$) and “it is four o’clock or life does not exist after death”($\alpha \lor \lnot \beta$). I later discover that my clock is broken. Then I must contract my belief $\alpha$ (but not revise by $\lnot \alpha$). According to *saturability* I must retain either $\alpha \lor \beta$ or $\alpha \lor \lnot \beta$, but I have no reason to do this.
Recovery $K \subseteq (K \land \alpha) + \alpha$

Example: I believed that I had my latchkey on me ($\alpha$). Then I felt in my left pocket, where I usually keep it, and did not find it. I lost my belief in $\alpha$ (but without starting to believe in $\neg \alpha$ instead). Half a second later, I found the key, and regained my belief in $\alpha$. 
Two Counterexamples …

**Example:** “I believe that “Cleopatra had a son” ($\alpha$) and that “Cleopatra had a daughter” ($\beta$), and thus also 'Cleopatra had a child’ ($\alpha \lor \beta$, briefly $\delta$). Then I receive information that make me give up my belief in $\delta$, and contract my belief set accordingly, forming $K-\delta$. Soon afterwards I learn from a reliable source that “Cleopatra had a child”. It seem perfectly reasonable for me to then add $\delta$ (i.e. $\alpha \lor \beta$) to my set of beliefs without also reintroducing either $\alpha$ or $\beta$.”

**Example:** “I previously entertained the two beliefs, “$x$ is divisible by 2” ($\alpha$) and “$x$ is divisible by 6” ($\beta$). When I received new information that induced me to give up the first of these beliefs ($\alpha$), the second ($\beta$) had to go as well (since $\alpha$ would otherwise follow from $\beta$). I then I received new information that made me accept the belief “$x$ is divisible by 8.” ($\epsilon$). Since $\alpha$ follows from $\epsilon$, $(K-\alpha) + \alpha$ is a subset of $(K-\alpha) + \epsilon$, then by recovery I obtain that “$x$ is divisible by 24” ($\delta$), contrary to the intuition.”
and one discussion

“as soon as contraction makes use of the notion 'y is believed only because of x', we run into counterexamples to recovery (...) But when a theory is taken as 'naked', i.e. as a bare set \( A=\text{Cn}(A) \) of statements closed under consequence, then recovery appears to be free of counterexamples.” (Makinson)

“Actual human belief always have such a justificatory structure (...). It is difficult if not impossible to find examples about which we can have intuitions, and in which the belief set is not associated with a justificatory structure that guides our intuitions. Against this background, it is not surprising that, as Makinson says, recovery “appears to be free of intuitive counterexamples” (...). It also seems to be free of confirming examples of the kind.” (Hansson)
Recovery in Axiomatic

Observation: (Niederee) Let $K$ be a belief set and $\alpha \in K$. Then, regardless of whether or not $\beta$ is in $K$, recovery together with closure implies that:

1. $\beta \rightarrow \alpha \in K - (\alpha \lor \beta)$.
2. $\alpha \in (K - (\alpha \lor \beta)) + \beta$
3. $\neg\beta \in (K - (\alpha \lor \beta)) + \neg\alpha$
Recovery in Axiomatic

Alternatives to Recovery

If $\beta \in K$ and $\beta \notin K - \alpha$ then

**Relevance:** there is some set $H$ such that $K - \alpha \subseteq H \subseteq K$ and $\alpha \notin H$ but $\alpha \in H + \beta$.

**Core Retainment:** there is some set $H$ such that $H \subseteq K$ and $\alpha \notin H$ but $\alpha \in H + \beta$.

**Disjunctive Elimination:** $\alpha \lor \beta \notin K - \alpha$.

Unfortunately, all of them are equivalent to recovery in the presence of the other postulates.
Recovery in Axiomatic

**Contraction**
- Closure
- Inclusion
- Vacuity
- Success
- Extensionality

**Revision**
- Closure
- Success
- Inclusion
- Vacuity
- Consistency
- Extensionality

**C. Factoring**
- Recovery

**D. Factoring**
- Levi Identity
Semantic of Recovery

\[ |K| \cap |\alpha| = |K - \alpha| \cap |\alpha| \]

(i.e., no \(\alpha\)-worlds are added in the contraction by \(\alpha\))
The Problem

*Revision success:* $\alpha \in K \ast \alpha$

*Contraction success:* If $\not\models \alpha$, then $\alpha \notin K \div \alpha$.

Several authors have found this to be an implausible feature.
The Problem

- Yesterday, the Pope called me to wish me good luck in the tutorial.
- Yesterday, my mother called me to wish me good luck in the tutorial.

- One day when you return back from work, your son tells you, as soon as you see him: "A dinosaur has broken grandma's vase in the living-room". You probably accept one part of the information, namely that the vase has been broken, while rejecting the part of it that refers to a dinosaur.

- Give up your belief in \( P \neq NP \), since you have not proof about it.
The Problem

“is totally trusting at each stage about the input information; it is willing to give up whatever elements of the background theory must be abandoned to render it consistent with the new information. Once this information has been incorporated, however, it is at once as susceptible to revision as anything else in the current theory.

Such a rule of revision seems to place an inordinate value on novelty, and its behaviour towards what it learns seems capricious”

Hence Cross and Thomason, 1992
Revision

There are many ways to construct *non prioritized* belief revision functions.

We can organize these ways into taxonomies

**Taxonomy based on how to revise**

We can classify the different models of belief revision according to the process that the revision function involves.

**Taxonomy based in the outcome of the revision**

We can classify the different models of belief revision according to the outcome of the belief revision process. This outcome can be reflected in the status of the revised belief, the change of the original belief set and the changes made in the epistemic state (that can be conditioned future changes).
**Taxonomy based on how to revise**

**Integrated Revision:** It consists in revising (or updating) the belief set in one single step.  
*Example of models that satisfy this condition:* AGM revision [AGM85], Updating [KM92].

**Decision + Revision:** It consists in a first step where it is decided if the input $\alpha$ is fully accepted, partially accepted or rejected and, in a second step, if $\alpha$ is not rejected, in which the belief set is revised by $\alpha$ or by the chosen part of $\alpha$.  
*Example of models that satisfy this condition:* Screened Revision [Mak97b], Selective Revision [FH99a].
Taxonomy based on how to revise

**Integrated Choice:** It consists in choosing among the originally believed sentences and the input $\alpha$ in one integrated step.

**Example of models that satisfy this condition:** Credibility-limited revision based on Epistemic Entrenchment and Possible world approach of Credibility-limited revision [HFCF01], Schlechta and Rabinowicz [Sch97, Rab95] revision, revision by comparison [FR04], improvement operators [KP08].

**Contraction + Expansion:** In these models revising consists in first contracting the belief state and then expanding the new state by the new belief.

**Example of models that satisfy this condition:** AGM revision via the Levi identity, internal revision of belief bases [Han93b].
**Taxonomy based on how to revise**

**Expansion + Contraction:** It consists in adding the new sentence to the corpus of belief and then contracting by the negation of the input sentence.

*Example of models that satisfy this condition:* External revision [Han93b].

**Expansion + Consolidation:** It consists in adding a new sentence to the corpus of belief and then regaining consistency.

*Example of models that satisfy this condition:* Semi-Revision [Han97a].
Taxonomy based in the outcome of the revision

**Non-Indifferent Revision Functions:** Basically, the new status of the input would be accepted or rejected i.e., the agent is forced to assume a strong epistemic attitude regarding the input sentence and cannot manifest ignorance about it. In symbols $\alpha \in K\alpha$ or $-\alpha \in K\alpha$.

**Example of models that satisfy this condition:** AGM revision, Updating, Screened Revision, Credibility-limited revision, Irrevocable Belief Revision, Internal Revision.
Taxonomy based in the outcome of the revision

**All:** The new input is accepted without any constraint: $\alpha \in K \circ \alpha$.

**Example of models that satisfy this condition:** AGM revision, internal revision, updating.

**All or Nothing:** Or the new input is accepted, or no change is made in the belief set: $\alpha \in K \circ \alpha$ or $K \circ \alpha = K$.

**Example of models that satisfy this condition:** Screened revision, Credibility-limited revision, Improvement operator (in one single step).
**Taxonomy based in the outcome of the revision**

**All or Less:** Either the input is accepted or the input induces a contraction process: $\alpha \in K \circ \alpha$ or $K \circ \alpha \subseteq K$.

**Example of models that satisfy this condition:** Semi-revision, Revision by Comparison.

**All or Inconsistency:** The new input is accepted, but under certain conditions the negation of the input still survive to the revision process: $\alpha \in K \circ \alpha$ or $K \circ \alpha = K_1$.

**Example of models that satisfy this condition:** AGM revision revised by an inconsistent sentence, Irrevocable belief revision, Revision by Comparison in the Collapsed Case.
Taxonomy based in the outcome of the revision

**Proxy success:** The input is not accepted, however other belief (typically weaker or a partial part of the input) is accepted: *There is some* $\beta$ *such that* $K \ast \alpha \vdash \beta, \vdash \alpha \rightarrow \beta$ *and* $K \ast \alpha = K \ast \beta$.

**Example of models that satisfy this condition:** Selective revision, Schlechta and Rabinowicz revision, Lin Revision [Lin96].

**Improvement:** This case occurs only in models with iteration, if the input is not accepted, improve its possibility to be accepted in later revisions: $\alpha \in K \circ \ldots \circ \alpha$.

**Example of models that satisfy this condition:** Improvement Operators.
**Example: Selective Revision**

**DEFINITION:** Let $K$ be a belief set, $*$ a *partial meet revision* for $K$ and $f$ a function from $\mathcal{L}$ to $\mathcal{L}$. The *selective revision* $\circ$, based on $*$ and $f$, is the operation such that for all sentences $\alpha$:

$$K \circ \alpha = K * f(\alpha)$$
Example: Selective Revision

Plausible properties for $f$:

implication $\vdash \alpha \rightarrow f(\alpha)$.

idempotence $\vdash f(f(\alpha)) \leftrightarrow f(\alpha)$.

extensionality If $\vdash \alpha \leftrightarrow \beta$ then $\vdash f(\alpha) \leftrightarrow f(\beta)$.

consistency preservation If $\nvdash \neg \alpha$, then $\nvdash \neg f(\alpha)$.

maximality $\vdash f(\alpha) \leftrightarrow \alpha$.

weak maximality If $K \nvdash \neg \alpha$, then $\vdash f(\alpha) \leftrightarrow \alpha$. 
Example: Selective Revision

**Weak success**  If $K \not\models -\alpha$, then $K \circ \alpha \vdash \alpha$.

**Proxy success**  There is a sentence $\beta$, such that $K \circ \alpha \vdash \beta$, $\vdash \alpha \rightarrow \beta$, and $K \circ \alpha = K \circ \beta$.

**Weak proxy success**  There is a sentence $\beta$, such that $K \circ \alpha \vdash \beta$ and $K \circ \alpha = K \circ \beta$.

**Consistent expansion**  If $K \not\models K \circ \alpha$ then $K \cup (K \circ \alpha) \vdash \bot$. 
Example: Selective Revision

\[ K \circ \alpha = K \ast f(\alpha) \]
Example: Selective Revision
Example: Selective Revision
Example: Selective Revision
Example: Credibility-Limited Revision

**DEFINITION:** Let $K$ be a belief set. The operation $\circ$ on $K$ is a credibility-limited revision on $K$ if and only if there is an AGM revision $\ast$ on $K$ and a set $\mathcal{C}$ of sentences such that for all sentences $\alpha$:

$$K \circ \alpha = \begin{cases} K \ast \alpha & \text{if } \alpha \in \mathcal{C} \\ K & \text{otherwise} \end{cases}$$
Example: Credibility-Limited Revision

Plausible properties for $\mathcal{C}$

Closure under Logical Equivalence  If $\vdash \alpha \leftrightarrow \beta$ and $\alpha \in \mathcal{C}$, then $\beta \in \mathcal{C}$.

Single sentence closure  If $\alpha \in \mathcal{C}$, then $Cn(\{\alpha\}) \subseteq \mathcal{C}$.

Disjunctive completeness  If $\alpha \lor \beta \in \mathcal{C}$, then either $\alpha \in \mathcal{C}$ or $\beta \in \mathcal{C}$.

Negation completeness  $\alpha \in \mathcal{C}$ or $\neg \alpha \in \mathcal{C}$.

Element consistency  If $\alpha \in \mathcal{C}$, then $\alpha \not\vdash \bot$.

Expansive credibility  If $K \not\vdash \alpha$, then $\neg \alpha \in \mathcal{C}$.
Example: Credibility-Limited Revision

Relative success \( \alpha \in K \circ \alpha \) or \( K \circ \alpha = K \).

Disjunctive success \( \alpha \in K \circ \alpha \) or \( \neg \alpha \in K \circ \alpha \).

Strict improvement If \( \alpha \in K \circ \alpha \) and \( \alpha \rightarrow \beta \), then \( \beta \in K \circ \beta \).

Regularity If \( \beta \in K \circ \alpha \) then \( \beta \in K \circ \beta \).

Strong regularity If \( \neg \beta \notin K \circ \alpha \) then \( \beta \in K \circ \beta \).

Strong consistency \( K \circ \alpha \neq K_{\perp} \).

Consistency preservation If \( K \neq K_{\perp} \) then \( K \circ \alpha \neq K_{\perp} \).

Disjunctive constancy If \( K \circ \alpha = K \circ \beta = K \) then \( K \circ (\alpha \lor \beta) = K \).

Consistent expansion If \( K \notin K \circ \alpha \) then \( K \cup (K \circ \alpha) \vdash \perp \).
Example: Credibility-Limited Revision
Example: Credibility-Limited Revision
Example: Credibility-Limited Revision
Example: Credibility-Limited Revision
**Example: Shielded Contraction**

**DEFINITION:** Let \( K \) be a belief set, \( - \) an AGM contraction operator on \( K \) and \( \mathcal{R} \) a subset of \( \mathcal{L} \) (the set of retractable sentences). Then \( \varpi \) is the *shielded AGM contraction* induced by \( - \) and \( \mathcal{R} \) if and only if:

\[
K \varpi \alpha = \begin{cases} 
K \setminus \alpha & \text{if } \alpha \in \mathcal{R} \\
K & \text{otherwise}
\end{cases}
\]
Example: Shielded Contraction

Plausible conditions for $\mathcal{R}$

**Conjunctive Completeness**  If $\alpha \land \beta \in \mathcal{R}$ then $\alpha \in \mathcal{R}$ or $\beta \in \mathcal{R}$.

**Non-Retractability Preservation**  $\mathcal{L} \setminus \mathcal{R} \subseteq K \ominus \alpha$.

**Non-Retractability Propagation**  If $\alpha \notin \mathcal{R}$, then $\text{Cn}(\{\alpha\}) \cap \mathcal{R} = \emptyset$. 
Example: Shielded Contraction

**Persistence**  If $K \in \vdash \beta \vdash \beta$, then $K \in \vdash \alpha \vdash \beta$.

**Relative Success**  $K \in \vdash \alpha = K$ or $K \in \vdash \alpha \not\vdash \alpha$.

**Conjunctive Constancy**  If $K \in \vdash \alpha = K \in \vdash \beta = K$ then $K \in \vdash (\alpha \land \beta) = K$.

**Success Propagation**  If $K \in \vdash \beta \vdash \beta$ and $\vdash \beta \to \alpha$ then $K \in \vdash \alpha \vdash \alpha$. 
Shielded Contraction vs Credibility-Limited Revision

Relation between $\mathcal{R}$ and $\mathcal{C}$

$\alpha \in \mathcal{R}$ if and only if $\neg \alpha \in \mathcal{C}$

Consistency-preserving Levi identity:

$$K \circ \alpha = \begin{cases} (K \oplus \neg \alpha) + \alpha & \text{if } K \oplus \neg \alpha \not\vdash \neg \alpha \\ K & \text{otherwise} \end{cases}$$

Harper Identity:

$$K \ominus \alpha = K \cap K \circ \neg \alpha$$
EXTENSIONS

BELIEF BASES
“in real life, when we perform a contraction or derogation, we never do it to the theory itself (in the sense of a set of propositions closed under consequence) but rather on some finite or recursive or at least recursively enumerable base for the theory” Makinson
Belief Bases

**Belief Base:** A set $B$ of formulas (usually finite).

**Epistemic attitudes:**
- $\alpha \in Cn(B)$: $\alpha$ (implicitly) believed.
- $\alpha \in B$: $\alpha$ explicitly believed.
- $\alpha \in Cn(B) \setminus B$: $\alpha$ merely derived.
Two Traditions in Belief Bases

à la Dalal: This case is associated with a coherentist epistemic representation in which the corpus of beliefs is considered as a whole and none of the parts has a structural feature that differentiates it from the others. Thus belief bases are a mere expressive resource, and the belief change operation use the whole theory to perform the change, i.e.; if $Cn(B_1) = Cn(B_2)$, then $Cn(B_1 - \alpha) = Cn(B_2 - \alpha)$. This principle is known as irrelevance of syntax.
Two Traditions in Belief Bases

à la Hansson: This case is associated with a foundacionist epistemic representation. In this case, the belief change is performed in the belief base.

Example:

\[ \alpha: \text{Paris is the capital of France.} \]

\[ \beta: \text{There is milk in the fridge.} \]

\[ \alpha, \beta \in B \Rightarrow \alpha \leftrightarrow \beta \in Cn(B) \]

Contract by \( \beta \) must give up \( \beta \) and \( \alpha \leftrightarrow \beta \).

In the rest of this part, we will assume Hansson’s approach.
"If \( \beta \) has been retracted from a base \( B \) in order to bar derivations of \( \alpha \) from \( B \), then the contraction of \( Cn(B) \) by \( \alpha \) should not contain any sentences which were in \( Cn(B) \) "just because" \( \beta \) was in \( Cn(B) \)." (Fuhrmann)
Why belief bases?

**Expressivity** \( B_1 = \{\alpha, \beta\}, \ B_2 = \{\alpha, \alpha \leftrightarrow \beta\} \).

\[
Cn(B_1) = Cn(B_2)
\]

\[
B_1 * \neg \alpha = \{\neg \alpha, \beta\}
\]

\[
B_2 * \neg \alpha = \{\neg \alpha, \alpha \leftrightarrow \beta\}
\]

\[
\beta \in Cn(B_1 * \neg \alpha), \ \text{but} \ \beta \notin Cn(B_2 * \neg \alpha).
\]

**Inconsistency Tolerance** \( B_1 = \{p, \neg p, q_1, q_2, q_3\} \)

\( B_2 = \{p, \neg p, \neg q_1, \neg q_2, \neg q_3\} \)

\[
Cn(B_1) = Cn(B_2), \ \text{but} \ Cn(B_1 \vdash \neg p) \neq Cn(B_2 \vdash \neg p)
\]
Why belief bases?

Implementations should use belief bases.

Define models with belief bases reduces the implementation gap.
The implementation Gap

The theoretical model

The implementation
How to construct belief bases functions?

Expansion: $B + \alpha = B \cup \{\alpha\}$.

Revision: $(B - \neg\alpha) \cup \{\alpha\}$ (via Levi identity).
How to construct belief bases functions?

**Contraction**: Current strategies:

- Replicate the constructive models of beliefs sets and obtain their axiomatic characterization in order to determine its “behavior”.

- Construct base functions from functions in belief sets.
Axioms: Simple Translations

**Success**  If $\not\vdash \alpha$, then $B-\alpha \not\vdash \alpha$.

**Inclusion**  $B-\alpha \subseteq B$.

(note that is different from $Cn(B-\alpha) \subseteq Cn(B)$)

**Vacuity**  If $A \not\vdash \alpha$, then $B \subseteq B-\alpha$.

**Conjunctive Factoring**  $B-\alpha \land \beta = \begin{cases} B-\alpha \text{ or} \\ B-\beta \text{ or} \\ B-\alpha \cap A-\beta \end{cases}$
Axioms: About sentences behaviour

**Extensionality**  If $\vdash \alpha \leftrightarrow \beta$, then $B - \alpha = B - \beta$

**Uniformity**  If it holds for all subsets $B'$ of $B$ that $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B - \alpha = B - \beta$. 
Axioms: Minimal Change

**Relevance** If $\beta \in B$ and $\beta \notin B - \alpha$ then there is some set $B'$ such that $B - \alpha \subseteq B'$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

**Core retention** If $\beta \in B$ and $\beta \notin B - \alpha$ then there is some set $B'$ such that $B' \subseteq B$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

**Weak Relevance** If $\beta \in B$ and $\beta \notin B - \alpha$ then there is some set $B'$ such that $B - \alpha \subseteq B' \subseteq Cn(B)$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

**Relative Closure** $B \cap Cn(B - \alpha) \subseteq B - \alpha$.

**Failure** (If $\vdash \alpha$, then $B - \alpha = B$)
Partial Meet Base Contraction

**Construction**

- $B_{\bot \alpha}$: maximal subsets of $B$ that fail to imply $\alpha$
- $\gamma$: function that selects some elements of $B_{\bot \alpha}$
- $B_{-\gamma \alpha} = \bigcap g(B_{\bot \alpha})$
Partial Meet Base Contraction

Axiomatic

Success  If $\not\models \alpha$, then $B-\alpha \not\models \alpha$.

Inclusion  $B-\alpha \subseteq B$.

  (note that is different from $Cn(B-\alpha) \subseteq Cn(B)$).

Uniformity  If it holds for all subsets $B'$ of $B$ that $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B-\alpha = B-\beta$.

Relevance  If $\beta \in B$ and $\beta \notin B-\alpha$ then there is some set $B'$ such that $B-\alpha \subseteq B' \subseteq B$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$. 
Kernel Base Contraction

Construction

- $B \parallel \alpha$: minimal subsets of $B$ that imply $\alpha$
- $\sigma$: function that selects at list one element of each set in $B \parallel \alpha$
- $B - \sigma \alpha = B \setminus \sigma(B \parallel \alpha)$
Kernel Base Contraction

Axiomatic

Success  If $\not\vdash \alpha$, then $B-\alpha \not\vdash \alpha$.

Inclusion  $B-\alpha \subseteq B$.

Uniformity  If it holds for all subsets $B'$ of $B$ that $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B - \alpha = B - \beta$.

Core retention  If $\beta \in B$ and $\beta \notin B-\alpha$ then there is some set $B'$ such that $B' \subseteq B$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$. 
Ensconcement

An ensconcement relation on a belief base $B$ is a transitive and connected relation $\leq$ that satisfies the following three conditions:

$(\leq 1)$ If $\beta \in B \setminus Cn(\emptyset)$, then $\{\alpha \in B : \beta < \alpha\} \not\models \beta$.

$(\leq 2)$ If $\not\models \alpha$ and $\vdash \beta$, then $\alpha < \beta$, for all $\alpha, \beta \in B$.

$(\leq 3)$ If $\vdash \alpha$ and $\vdash \beta$, then $\alpha \leq \beta$, for all $\alpha, \beta \in B$. 

Created by Mary-Anne Williams inspired in Epistemic Entrenchment
Ensconce\ment

\[ \text{cut}_B(\alpha) = \{ \beta \in B : \{ \gamma \in B : \beta \leq \gamma \} \not\models \alpha \} \]

- is an Ensconce\ment-Based Contraction Function if and only if:

\[ \beta \in B - \alpha \text{ if and only if } \beta \in B \text{ and either} \]

(i) \( \alpha \in Cn(\emptyset) \)

\[ \text{or} \]

(ii) \( \text{cut}_B(\alpha) \vdash \alpha \lor \beta \)
Ensconnement

Axiomatic

Success If $\not\models \alpha$, then $B - \alpha \not\models \alpha$.

Inclusion $B - \alpha \subseteq B$.

Extensionality If $\models \alpha \iff \beta$, then $B - \alpha = B - \beta$.

Weak Relevance If $\beta \in B$ and $\beta \notin B - \alpha$ then there is some set $B'$ such that $B - \alpha \subseteq B' \subseteq \text{cn}(B)$ and $\alpha \notin \text{cn}(B')$ but $\alpha \in \text{cn}(B' \cup \{\beta\})$.

Conjunctive Factoring $B - \alpha \land \beta = \begin{cases} B - \alpha & \text{or} \\ B - \beta & \text{or} \\ B - \alpha \cap A - \beta & \end{cases}$
Basic Related AGM-Contraction

\[ B - \alpha = \left( Cn(B) \div \alpha \right) \cap B \]

**Axiomatic**

**Success**  If \( \not\models \alpha \), then \( B - \alpha \not\models \alpha \).

**Inclusion**  \( B - \alpha \subseteq B \).

**Extensionality**  If \( \vdash \alpha \leftrightarrow \beta \), then \( B - \alpha = B - \beta \)

**Weak Relevance**  If \( \beta \in B \) and \( \beta \notin B - \alpha \) then there is some set \( B' \) such that \( B - \alpha \subseteq B' \subseteq Cn(B) \) and \( \alpha \notin Cn(B') \) but \( \alpha \in Cn(B' \cup \{\beta\}) \).
The interconnection

- Partial Meet Contraction
- Smooth Kernel Contraction
- Kernel Contraction

Postulates
- FFKI06
- Relevance

Kernel Contraction with relevance
- Basic related-AGM base contraction

Relative closure
- Core - Retainment
- Uniformity
- Weak Relevance
- Vacuity
- Success
- Extensionality
- Inclusion

Han91
Han94
Challenges

- Complete the theoretical models
- Understand better the concept of minimal change.
- Understand how the model works at the supplementary level.
- Implement (outside the toy problems) belief base change for resource-bounded agents.
EXTENSIONS

ITERATED MODELS
An AGM contraction or revision takes us from a belief set to a new belief set.
An AGM contraction or revision takes us from a belief set to a new belief set.

\[ K \alpha \]
Iteration

However, it does not provide a new selection mechanism to be used for further changes of the new belief set.
Iteration

However, it does not provide a new selection mechanism to be used for further changes of the new belief set.
The problem of constructing models that allow for iterated change is probably the most studied problem in the literature on belief change.
Belief States

Furthermore, the operation of change has to yield a complete such belief state representation as its outcome, not merely a new belief set.

There are several ways to represent such an extended epistemic state. The most common of these is a preorder on the set of possible worlds, or equivalently a complete sphere system.

The belief set can be inferred from this preorder; it is simply the intersection of the worlds in the highest equivalence class (innermost sphere).

An operation of change gives rise to a new preorder (sphere system), from which the new belief set can be inferred, and which can in its turn be subject to further changes.
Darwiche and Pearl modified the list of KM postulates to work in the more general framework of epistemic states:

(R*1) \( B(\psi \circ \alpha) \vdash \alpha \)

(R*2) If \( B(\psi) \land \alpha \nvdash \bot \) then \( B(\psi \circ \alpha) \equiv \varphi \land \alpha \)

(R*3) If \( \alpha \nvdash \bot \) then \( B(\psi \circ \alpha) \nvdash \bot \)

(R*4) If \( \psi_1 = \psi_2 \) and \( \alpha_1 \equiv \alpha_2 \) then \( B(\psi_1 \circ \alpha_1) \equiv B(\psi_2 \circ \alpha_2) \)

(R*5) \( B(\psi \circ \alpha) \land \psi \vdash B(\psi \circ (\alpha \land \psi)) \)

(R*6) If \( B(\psi \circ \alpha) \land \psi \nvdash \bot \) then \( B(\psi \circ (\alpha \land \psi)) \vdash B(\psi \circ \alpha) \land \psi \)
Major Classes of Iterated Operators

Hans Rott has identified at least Twenty-Seven Iterated Theory Change Operators.
Major Classes of Iterated Operators

We can divide iterable operators into three classes according to their ability to remember and to take the revision history into account:

Operators with full memory: In this case the full history of changes is conserved, so that rollbacks of previous changes are possible.
Major Classes of Iterated Operators

We can divide iterable operators into three classes according to their ability to remember and to take the revision history into account:

Operators without memory: In this case, each belief set is revised in a predetermined way, independently of how it was obtained:

If $\Psi \circ \alpha$ and $\Upsilon \circ \alpha$ have the same belief set, then so have $\Psi \circ \alpha \circ \beta$ and $\Upsilon \circ \alpha \circ \beta$. 
Major Classes of Iterated Operators

We can divide iterable operators into three classes according to their ability to remember and to take the revision history into account:

Operators with partial memory: In this case it makes a difference for future revisions how a belief set was arrived at, but the information remembered is not sufficient to identify the previous states. Most of the proposed iterable revision operators are of this type.
Darwiche and Pearl proposed the following conditions for iteration:

(DP1) If \( q \models \alpha \), then \( (\Psi \circ \alpha) \circ \beta = \Psi \circ \beta \).

(DP2) If \( q \models \neg \alpha \), then \( (\Psi \circ \alpha) \circ \beta = \Psi \circ \beta \).

(DP3) If \( \Psi \circ \beta \models \alpha \), then \( (\Psi \circ \alpha) \circ \beta \models \alpha \).

(DP4) If \( \Psi \circ \beta \not\models \neg \alpha \), then \( (\Psi \circ \alpha) \circ \beta \not\models \neg \alpha \).

\( (DP1) \): For any \( \mu \models \alpha \) and \( \phi \models \alpha \), \( \mu \leq_{\Psi} \phi \) iff \( \mu \leq_{\Psi \circ \alpha} \phi \).

\( (DP2) \): For any \( \mu \models \neg \alpha \) and \( \phi \models \neg \alpha \), \( \mu \leq_{\Psi} \phi \) iff \( \mu \leq_{\Psi \circ \alpha} \phi \).

\( (DP3) \): For any \( \mu \models \alpha \) and \( \phi \models \neg \alpha \), if \( \mu \prec_{\Psi} \phi \), then \( \mu \prec_{\Psi \circ \alpha} \phi \).

\( (DP4) \): For any \( \mu \models \alpha \) and \( \phi \models \neg \alpha \), if \( \mu \leq_{\Psi} \phi \), then \( \mu \leq_{\Psi \circ \alpha} \phi \).
Jin & ThIELscher and Booth & Meyer have pointed out that these postulates are too permissive.

They proposed the following additional condition:

\[(Ind): \text{For any } \mu \models \alpha \text{ and } \phi \models \neg \alpha, \text{ if } \mu \preceq \psi \phi, \text{ then } \mu \preceq_{\psi \circ \alpha} \phi.\]
Three Classes of Operators with Partial Memory

Conservative revision, originally called natural revision, has been studied by Boutilier. This operation is conservative in the sense that it only makes the minimal changes of the preorder that are needed to accept the input.

In revision by $\alpha$, the maximal $\alpha$-worlds are moved to the top of the preorder which is otherwise left unchanged.

(Nat): If $\mu \notin [\Psi \circ \alpha]$ and $\phi \notin [\Psi \circ \alpha]$, then $\mu \leq_{\Psi} \phi$ iff $\mu \leq_{\Psi \circ \alpha} \phi$. 
Three Classes of Operators with Partial Memory

*Moderate revision*, also called lexicographic revision, was originally studied by Nayak. When revising by $\alpha$ it rearranges the preorder by putting the $\alpha$-worlds at top (but conserving their relative order) and the $\neg\alpha$-worlds at bottom (but conserving their relative order).

\[\text{(Lex): If } \mu \models \alpha \text{ and } \phi \models \neg\alpha, \text{ then } \mu \prec_{\psi_{\alpha,\alpha}} \phi.\]
Three Classes of Operators with Partial Memory

Radical revision Proposed by Segerberg. Is similar to moderate revision, but it differs in making the new belief irrevocable, i.e., impossible to remove.

In radical revision by $\alpha$, the relative order of the $\alpha$-worlds is retained whereas the $\neg\alpha$-worlds are removed from the preorder, thus becoming inaccessible.

(Irr): $[(\Psi \circ \alpha) \circ \neg\alpha] = \emptyset$. 
Alternative Models

- Update
- Merging
- Improvement Operators
- Others: Consolidation, Semi-Revision, External Revision, CLIO.
In 1992, Katsuno and Mendelzon presented a type of operator of change that they called update. Whereas revision operators are intended to capture the change yielded by evolving knowledge about a static situation, update operators are intended to mirror the change in knowledge produced by an evolving situation.

“We make a fundamental distinction between two kinds of modifications to a knowledge base. The first one, update consists of bringing the knowledge base up to date, when the world described by it changes. ... The second kind of modification, revision, is used when we are obtaining new information about a static world. ... We claim the AGM postulates describe only revisions.”
Update

There are important formal differences between update and AGM revision; in particular the AGM postulate Vacuity (If $K \not\vdash \neg \alpha$ then $K + \alpha \subseteq K * \alpha$) does not hold for updates.

(U1) $\varphi \circ \mu \vdash \mu$

(U2) If $\varphi \vdash \mu$, then $\varphi \circ \mu \equiv \varphi$

(U3) If $\varphi$ is consistent and $\mu$ is consistent, then $\varphi \circ \mu$ is consistent

(U4) If $\varphi_1 \equiv \varphi_2$ and $\mu_1 \equiv \mu_2$ then $\varphi_1 \circ \mu_1 \equiv \varphi_2 \circ \mu_2$

(U5) $(\varphi \circ \mu) \land \phi \vdash \varphi \circ (\mu \land \phi)$

(U6) If $\varphi \circ \mu_1 \vdash \mu_2$ and $\varphi \circ \mu_2 \vdash \mu_1$, then $\varphi \circ \mu_1 \equiv \varphi \circ \mu_2$

(U7) If $\varphi$ is a complete formula then $(\varphi \circ \mu_1) \land (\varphi \circ \mu_2) \vdash \varphi \circ (\mu_1 \lor \mu_2)$

(U8) $(\varphi_1 \lor \varphi_2) \circ \mu \equiv (\varphi_1 \circ \mu)) \lor (\varphi_2 \circ \mu)$
An update operator $\diamond$ satisfies (U1)-(U8) if and only if there exists a faithful assignment that maps each interpretation $\omega$ to a partial pre-order $\leq_\omega$ and such that

$$\text{mod } \varphi \diamond \mu = \bigcup_{\omega=\varphi} \min(\text{mod } \mu, \leq_\omega)$$
Update
Update
Update
Merging

It was proposed by Konieczny and Pino Perez.
Belief merging is the process of defining the beliefs of a group of agents. Given a set of agents that have their own beliefs, what can be considered as the beliefs of the group?

\[
\begin{align*}
\varphi_1 & : a, b \rightarrow c \\
\varphi_2 & : a, b \\
\varphi_3 & : \neg a
\end{align*}
\]

\[
\Delta(\{\varphi_1, \varphi_2, \varphi_3\}) = b \rightarrow c, b, a
\]
Merging

(IC0) \( \triangle_{\mu}(\Psi) \models \mu \)

(IC1) If \( \mu \) is consistent, then \( \triangle_{\mu}(\Psi) \) is consistent

(IC2) If \( \wedge \Psi \) is consistent with \( \mu \), then \( \triangle_{\mu}(\Psi) \equiv \wedge \Psi \wedge \mu \)

(IC3) If \( \Psi_1 \equiv \Psi_2 \) and \( \mu_1 \equiv \mu_2 \), then \( \triangle_{\mu_1}(\Psi_1) \equiv \triangle_{\mu_2}(\Psi_2) \)

(IC4) If \( \varphi_1 \models \mu \) and \( \varphi_2 \models \mu \), then \( \triangle_{\mu}(\{\varphi_1, \varphi_2\}) \wedge \varphi_1 \) is consistent if and only if \( \triangle_{\mu}(\{\varphi_1, \varphi_2\}) \wedge \varphi_2 \) is consistent

(IC5) \( \triangle_{\mu}(\Psi_1) \wedge \triangle_{\mu}(\Psi_2) \models \triangle_{\mu}(\Psi_1 \sqcup \Psi_2) \)

(IC6) If \( \triangle_{\mu}(\Psi_1) \wedge \triangle_{\mu}(\Psi_2) \) is consistent, then
\[ \triangle_{\mu}(\Psi_1 \sqcup \Psi_2) \models \triangle_{\mu}(\Psi_1) \wedge \triangle_{\mu}(\Psi_2) \]

(IC7) \( \triangle_{\mu_1}(\Psi) \wedge \mu_2 \models \triangle_{\mu_1 \wedge \mu_2}(\Psi) \)

(IC8) If \( \triangle_{\mu_1}(\Psi) \wedge \mu_2 \) is consistent, then \( \triangle_{\mu_1 \wedge \mu_2}(\Psi) \models \triangle_{\mu_1}(\Psi) \)
Merging
Merging

\[ \varphi_1, \varphi_2, \varphi_3 \]
Merging

\[ \Delta(\{\varphi_1, \varphi_2, \varphi_3\}) \]
Also it was proposed by Konieczny and Pino Perez

- Weak primacy of update.
- The plausibility of the new information must be increased after the improvement.
- This can facilitate the acceptance of $\alpha$ in later, additional operations, so that we can have $\varphi \circ \alpha \not\vdash \alpha$, but $\varphi \circ \alpha \circ \alpha \vdash \alpha$.
- $(Imp)$: Let $\mu \models \alpha$ and $\phi \models \neg \alpha$. If $\phi \ll \psi \mu$, then $\mu \leq_{\psi \alpha} \phi$. 
Improvement

(I1) There exists $n$ such that $\psi \circ^n \alpha \vdash \alpha$

(I2) If $\psi \wedge \alpha \not\vdash \bot$, then $\psi \star \alpha \equiv \psi \wedge \alpha$

(I3) If $\alpha \not\vdash \bot$, then $\psi \circ \alpha \not\vdash \bot$

(I4) For any positive integer $n$ if $\alpha_i \equiv \beta_i$ for all $i \leq n$ then

\[ \psi \circ \alpha_1 \circ \cdots \circ \alpha_n \equiv \psi \circ \beta_1 \circ \cdots \circ \beta_n \]

(I5) $(\psi \star \alpha) \wedge \beta \vdash \psi \ast (\alpha \wedge \beta)$

(I6) If $(\psi \star \alpha) \wedge \beta \not\vdash \bot$, then $\psi \ast (\alpha \wedge \beta) \vdash (\psi \star \alpha) \wedge \beta$

(I7) If $\alpha \vdash \beta$ then $(\psi \circ \beta) \star \alpha \equiv \psi \star \alpha$

(I8) If $\alpha \vdash \neg \beta$ then $(\psi \circ \beta) \ast \alpha \equiv \psi \star \alpha$

(I9) If $\psi \star \alpha \not\vdash \neg \beta$ then $(\psi \circ \beta \ast \alpha) \vdash \beta$

(I10) If $\psi \star \alpha \vdash \neg \beta$ then $(\psi \circ \beta \ast \alpha) \not\vdash \beta$

(I11) If $\psi \star \alpha \vdash \neg \beta$, $\alpha \wedge \beta \not\vdash \bot$ and $\alpha \ll \psi \alpha \wedge \beta$

\[ \text{then } (\psi \circ \beta) \ast \alpha \not\vdash \neg \beta \]
Improvement
Improvement
Improvement
Improvement
CLIO (Credibility-Limited Improvement Operators)
Belief Bases Models

Belief Bases can recognize between different inconsistent bases:
\[ B_1 = \{ \alpha, -\alpha \} \]
\[ B_2 = \{ \alpha, \beta, \alpha \rightarrow \neg \beta \} \]
then new operators can be defined for inconsistent belief bases.
Hansson proposed:

**Consolidation**  Makes a belief base consistent.
\[ B! = B_{-1} \]

**Semi-Revision**  Non-prioritized revision, new info may be rejected.
\[ B?\alpha = (B + \alpha)! \]

**External Revision**  (Reversed Levi Identity)
\[ B \pm \alpha = (B + \alpha) - \neg \alpha \]
PERSPECTIVES

Hot Topics
Hot topics

• Ontologies
• Horn Contraction
• Argumentation
• Implementations
• … (incomplete list)
Ontologies

- An ontology in computer science is an explicit, formal specification of the terms of a domain of application, along with the relations among these terms.
- An ontology provides a (structured) vocabulary which forms the basis for the representation of general knowledge.
- Ontologies have found extensive application in Artificial Intelligence and the Semantic Web, as well as in areas such as software engineering, bioinformatics, and database systems.
Ontologies: Description Logics

Research in ontologies in Artificial Intelligence has focussed on description logics (DL), a (decidable) fragment of first order logic.

Two components,

**TBox**, for expressing concepts.
Characterises a domain of application.

**ABox**, that contains assertions about specific individuals and instances.
Contains information on a specific instance of a domain
Crucially, an ontology will be expected to evolve,

- either as domain information is corrected and refined, or
- in response to a change in the underlying domain.

In a description logic, such change may come in two different forms:

- the background knowledge, traditionally stored in the TBox, may require modification, or
- the ground facts or data, traditionally stored in the ABox, may be modified.
Horn Belief Change

Address belief change in the expressively weaker language of *Horn clauses*, where a Horn clause can be written as a rule in the form \( a_1 \land a_2 \land \cdots \land a_n \rightarrow a \) for \( n \geq 0 \), and where \( a, a_i \) (\( 1 \leq i \leq n \)) are atoms.

An agent’s beliefs are represented by a Horn clause knowledge base, and the input is a conjunction of Horn clauses.
Horn Belief Change

This topic is interesting for several reasons. It sheds light on the theoretical underpinnings of belief change, in that it weakens the assumption that the underlying logic contains propositional logic. As well, Horn clauses have found extensive use in artificial intelligence and database theory, in areas such as logic programming, truth maintenance systems, and deductive databases.
Argumentation deals with strategies agents employ for their own reasoning or to change the beliefs of other agents.

**Argument**: A set of statements, composed of three parts: a set of premises, a conclusion, and an inference.

**Defeasibility**: When a conclusion is defeated by new information, that reasoning is defeasible.

**The attack/refutation question**: Arguments can attack or support other arguments.

**Argumentative Systems**: A way to formalize common-sense reasoning. Formalisms that intend to support Argumentation and standardize it.
Argumentation

Dung’s Abstract Framework

- Abstract notion of argument and big possibility of extension
- Argumentation framework: $AF \langle \text{Arg}, \text{Attacks} \rangle$
- Attack relation between arguments: An argument $A$ will be defeated if it is possible to find at least one defeater for it that is not defeated
Argumentation

Different ways of applying BR in Argumentation

- Changing by adding or deleting an argument or a set of them.
- Changing the attack (defeat) relation among arguments.
- Changing the status of beliefs.
- Changing the type of an argument (strict to defeasible, or vice versa).
Implementations

- Actually there exists very few “real” implementations.
- Is the future of Belief Revision in Computer Science (and its big challenge !).
- It will require to combine different models:
  - Multi-agents systems
  - Horn Belief Change
  - Argumentations (by means of programs like DeLP Defeasible Logic Programming)
  - Belief Bases
  - ...
CONCLUSIONS
Conclusions

• This tutorial was based on

“AGM 25 Years: Twenty-Five Years of Research in Belief Change”
Eduardo Fermé and Sven Ove Hansson.
Journal of Philosophical Logic 40, (2) : 113-114. 2011

... plus some updates
Conclusions

• This tutorial was an attempt to summarize (in 4 hours) major developments in twenty-eight years of AGM-inspired research.

• In preparing the paper we benefited from the help of more than fifty colleagues who answered our queries and provided us with information.

• I also received many suggestions and material from friends and colleagues that help me to improve the presentation.

• In addition to its academic excellence, the belief revision community is a remarkably generous one.

• This bodes well for the future of belief revision research.
Thanks!

Questions?

The End