Answer Set Solving in Practice

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Rough Roadmap

1. Introduction
2. Language
3. Modeling
4. Grounding
5. Foundations
6. Solving
7. Systems
8. Applications

Resources

- Course material
  - http://www.cs.uni-potsdam.de/wv/lehre
  - http://moodle.cs.uni-potsdam.de
  - http://potassco.sourceforge.net/teaching.html
- Systems
  - clasp http://potassco.sourceforge.net
  - dlv http://www.dlvsystem.com
  - smodels http://www.tcs.hut.fi/Software/smodels

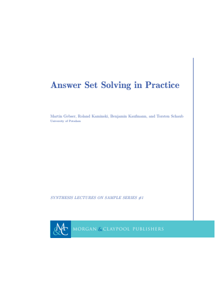
  - gringo http://potassco.sourceforge.net
  - lparse http://www.tcs.hut.fi/Software/smodels

  - clingo http://potassco.sourceforge.net
  - iclingo http://potassco.sourceforge.net
  - oclingo http://potassco.sourceforge.net

  - asparagus http://asparagus.cs.uni-potsdam.de

The (forthcoming) Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- http://potassco.sourceforge.net/book.html
- http://potassco.sourceforge.net/teaching.html

Literature

Books [4], [29], [53]
Surveys [50], [2], [39], [21], [11]
Articles [41], [42], [6], [61], [54], [49], [40], etc.

Motivation: Overview

1 Motivation

2 Nutshell

3 Shifting paradigms

4 Rooting ASP

5 ASP solving

6 Using ASP

Overview

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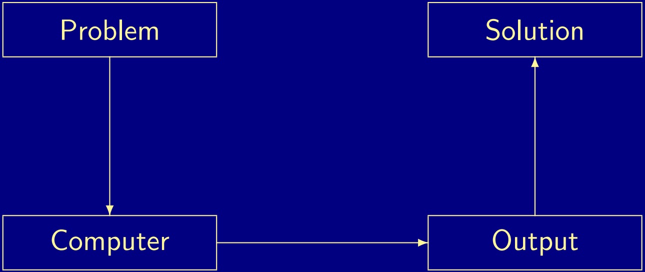
Informatics

"What is the problem?" versus *"How to solve the problem?"*



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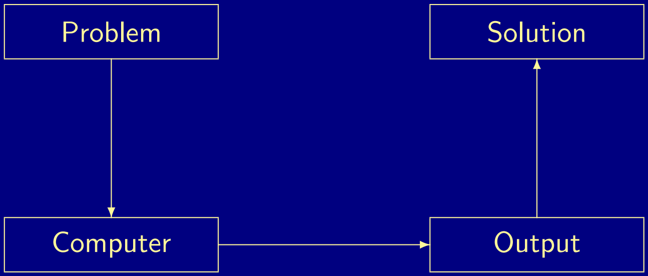

Traditional programming

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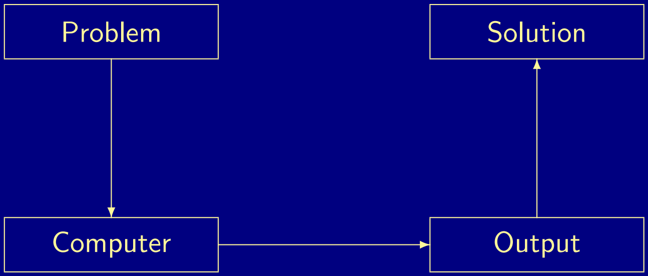

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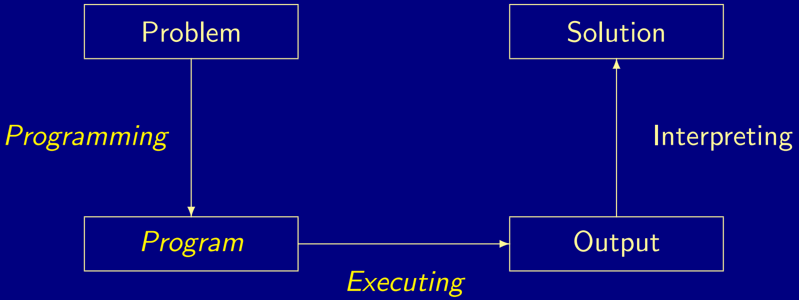

Declarative problem solving

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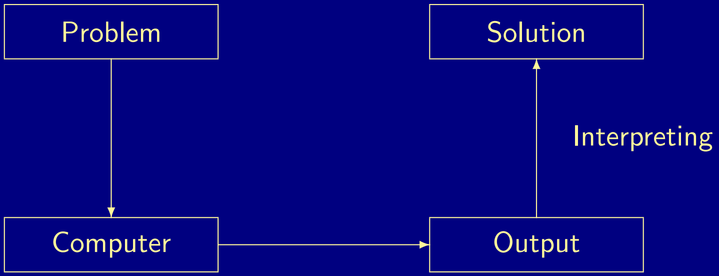

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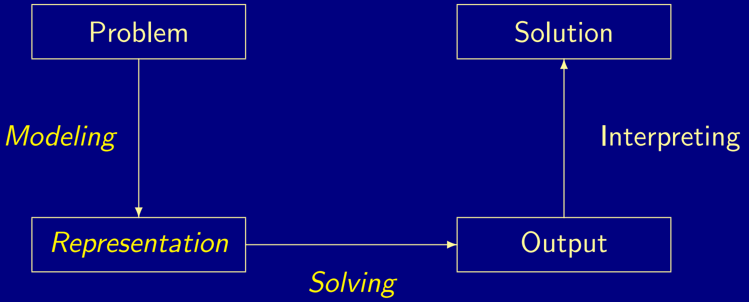

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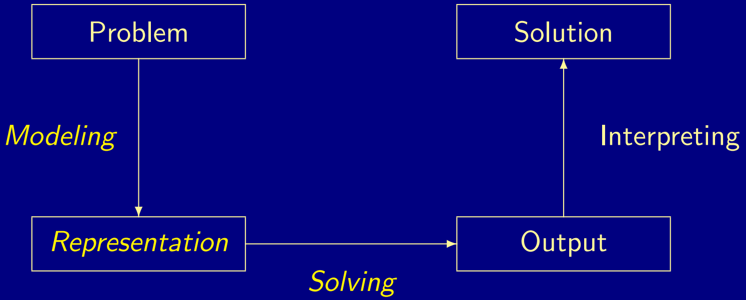

Overview

1 Motivation

2 Nutshell

3 Shifting paradigms

4 Rooting ASP

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6 Using ASP

Answer Set Programming

in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way
- ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas

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Answer Set Programming

in a Hazelnutshell

- ASP is an approach to declarative problem solving, combining
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  tailored to Knowledge Representation and Reasoning

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SAT}$$

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KR's shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem
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Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Model Generation based Problem Solving

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first-order theories	Herbrand models
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LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Prolog queries (testing entailment)

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?- above(a,c).
true.

?- above(c,a).
no.
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LP-style playing with blocks

Shuffled Prolog program

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on(a,b).
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above(X,Y) :- on(X,Y).
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Prolog queries

```
?- above(a,c).

Fatal Error: local stack overflow.
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Prolog queries (answered via fixed execution)

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?- above(a,c).

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SAT-style playing with blocks

Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\wedge & on(b,c) \\
\wedge & (on(X,Y) \rightarrow above(X,Y)) \\
\wedge & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y))
\end{array}
$$

Herbrand model

$$
\left\{ \begin{array}{ccccc}
on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\
above(a,b), & above(b,c), & above(a,c), & above(b,b), & above(c,b)
\end{array} \right\}
$$

SAT-style playing with blocks

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\begin{array}{ll}
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Herbrand model (among 426!)

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\right\}
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SAT-style playing with blocks

Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\land & on(b,c) \\
\land & (on(X,Y) \rightarrow above(X,Y)) \\
\land & (on(X,Z) \land above(Z,Y) \rightarrow above(X,Y))
\end{array}
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➥ Answer Set Programming (ASP)

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Answer Set Programming *at large*

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Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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ASP-style playing with blocks

Logic program

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Stable Herbrand model

$\{ \texttt{on}(a,b),\ \texttt{on}(b,c),\ \texttt{above}(b,c),\ \texttt{above}(a,b),\ \texttt{above}(a,c) \}$

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ASP versus LP

ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing +) $NP(^{NP})$	Turing

ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Optimization	—
Intersection/Union	—
(Turing +) $NP(^{NP})$	NP

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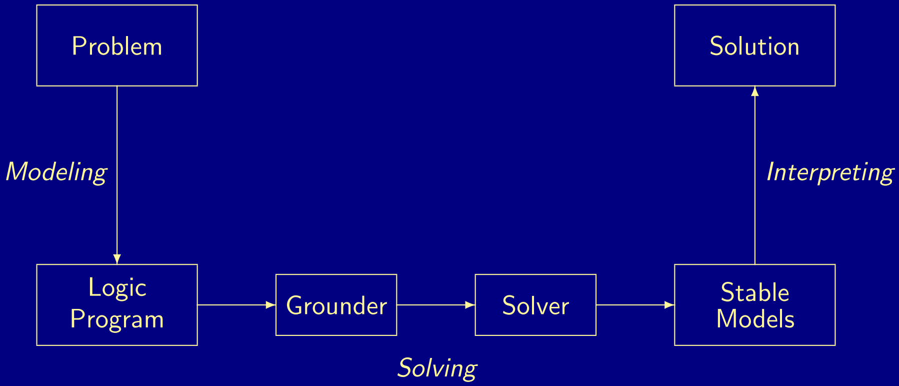
Problem
Solution
Modeling
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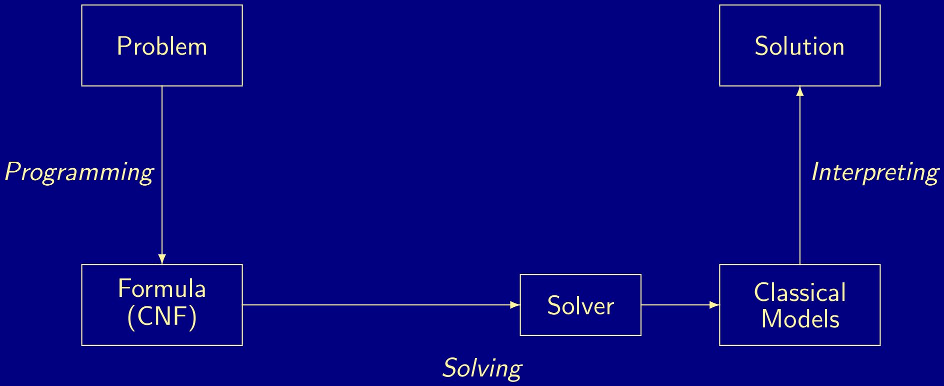
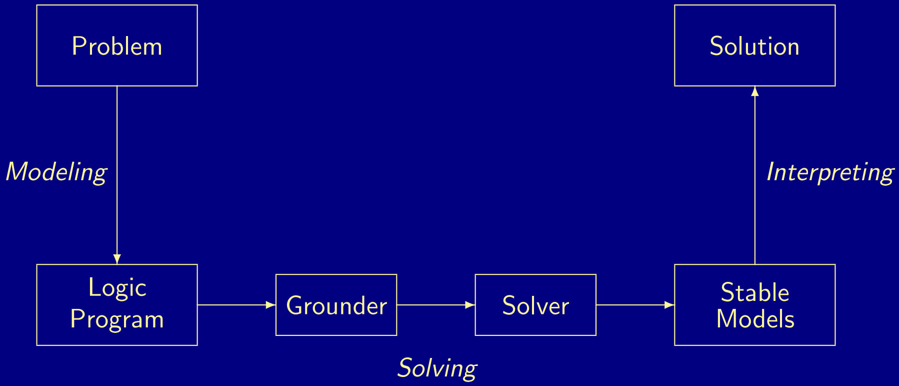
Logic Program
Grounder
Solver
Stable Models
Solving

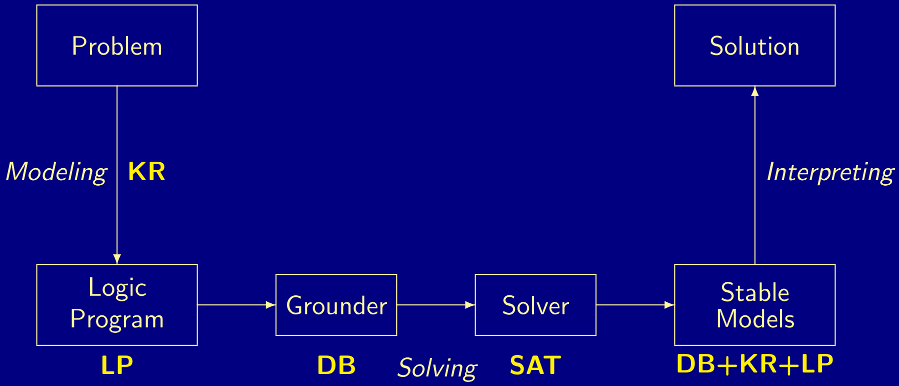
SAT solving


Problem
Solution
Programming
Interpreting
Formula (CNF)
Solver
Classical Models
Solving

Rooting ASP solving



Rooting ASP solving



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6 Using ASP

Two sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - System Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more

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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc

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ASP = DB+LP+KR+SAT

Introduction: Overview

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8 Semantics

9 Examples

10 Variables

11 Language constructs

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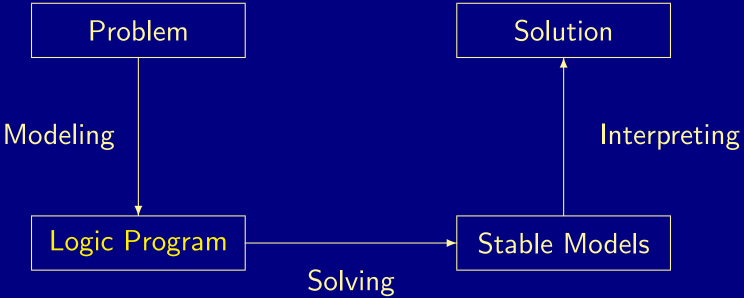
Problem solving in ASP: Syntax



Normal logic programs

- A (normal) logic program over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

$$\begin{aligned} head(r) &= a_0 \\ body(r) &= \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\ body(r)^+ &= \{a_1, \dots, a_m\} \\ body(r)^- &= \{a_{m+1}, \dots, a_n\} \end{aligned}$$

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- Notation

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Rough notational convention

We sometimes use the following notation interchangeably
in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`|`		`not`	`-`
logic program		$\leftarrow$	$,$	$;$		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

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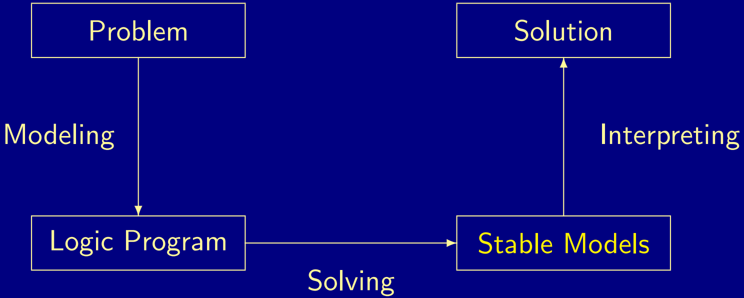
Problem solving in ASP: Semantics



Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Some "logical" remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

$$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model
- Horn clauses are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program P, $Cn(P)$ corresponds to the smallest model of the set of definite clauses corresponding to P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Φ $\boxed{q \;\wedge\; (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

P_Φ
$$\boxed{\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \ \sim r \end{array}}$$

Informally, a set X of atoms is a stable model of a logic program P

- if X is a (classical) model of P and
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$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Note: $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X
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A closer look at P^X

- In other words, given a set X of atoms from P,

  P^X is obtained from P by deleting
  1. each rule having $\sim a$ in its body with $a \in X$
     and then
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A first example

$P = \{p \leftarrow p, \; q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
$\emptyset$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

A first example

$$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$$

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$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✓
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A second example

$P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$

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A third example

$P = \{p \leftarrow \sim p\}$

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Some properties

- A logic program may have zero, one, or multiple stable models!
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Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \to \mathcal{T}, var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

Programs with Variables

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- Ground Instantiation of P: $\quad ground(P) = \bigcup_{r \in P} ground(r)$

An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

$$ground(P) = \left\{ \begin{array}{llllll} r(a,b) \leftarrow , & & & & & \\ r(b,c) \leftarrow , & & & & & \\ t(a,a) \leftarrow & r(a,a), & t(b,a) \leftarrow & r(b,a), & t(c,a) \leftarrow & r(c,a), \\ t(a,b) \leftarrow & r(a,b), & t(b,b) \leftarrow & r(b,b), & t(c,b) \leftarrow & r(c,b), \\ t(a,c) \leftarrow & r(a,c), & t(b,c) \leftarrow & r(b,c), & t(c,c) \leftarrow & r(c,c) \end{array} \right\}$$

Intelligent Grounding aims at reducing the ground instantiation

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Stable models of programs with Variables

Let P be a normal logic program with variables

- A set X of (ground) atoms as a stable model of P,
  if $Cn(ground(P)^X) = X$

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Overview

7 Syntax

8 Semantics

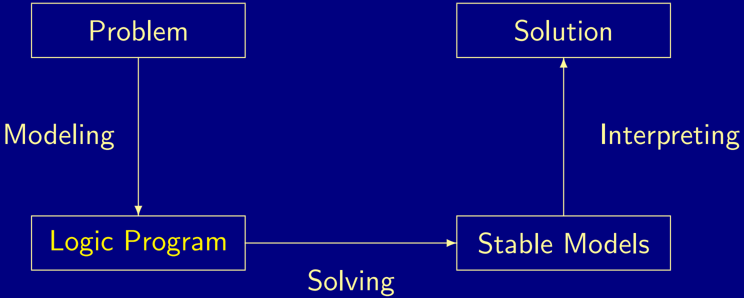
9 Examples

10 Variables

11 Language constructs

12 Reasoning modes

Problem solving in ASP: Extended Syntax



Language Constructs

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  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
    `p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)`
- Conditional Literals
  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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- Disjunction
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- Integrity Constraints
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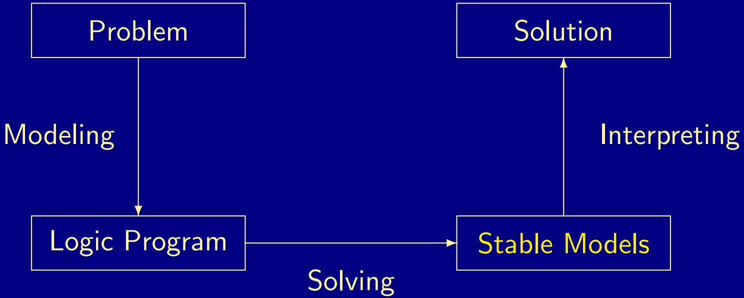
12 Reasoning modes

Problem solving in ASP: Reasoning Modes



Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

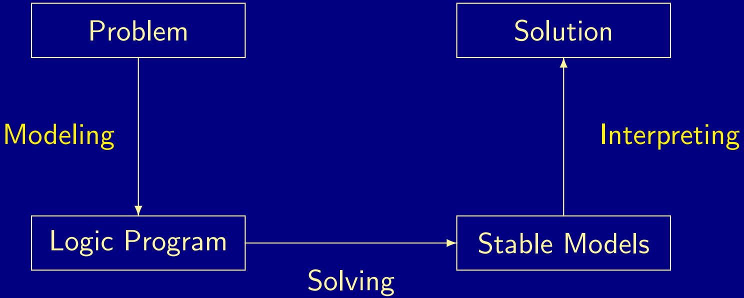
‡ without solution enumeration

Basic Modeling: Overview

13 ASP solving process

14 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Modeling and Interpreting



Modeling

- For solving a problem class **C** for a problem instance **I**, encode
  1. the problem instance **I** as a set $P_{\mathbf{I}}$ of facts and
  2. the problem class **C** as a set $P_{\mathbf{C}}$ of rules

  such that the solutions to **C** for **I** can be (polynomially) extracted from the stable models of $P_{\mathbf{I}} \cup P_{\mathbf{C}}$
- $P_{\mathbf{I}}$ is (still) called problem instance
- $P_{\mathbf{C}}$ is often called the problem encoding
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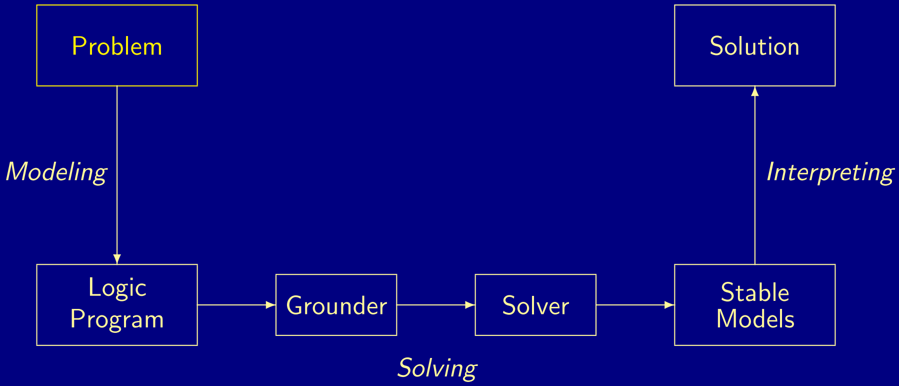
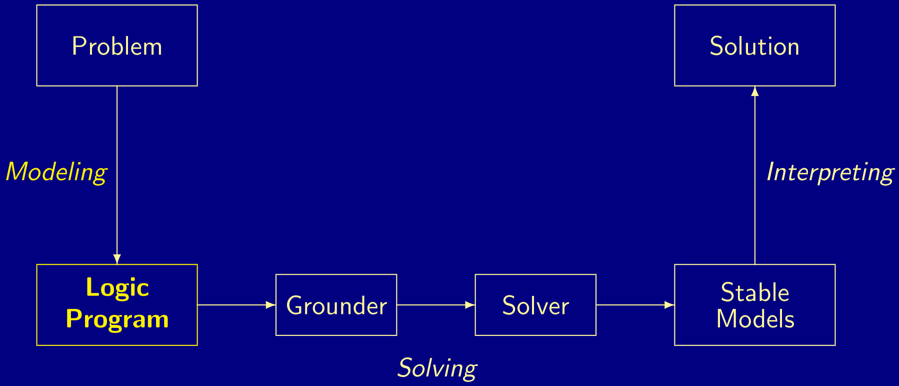
ASP solving process



ASP solving process



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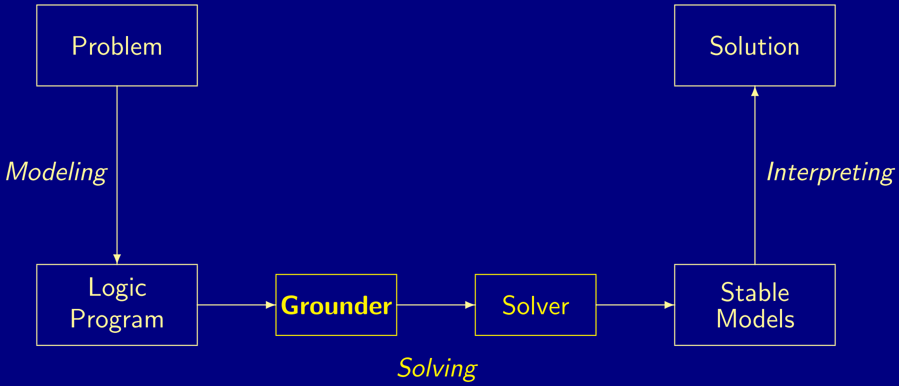

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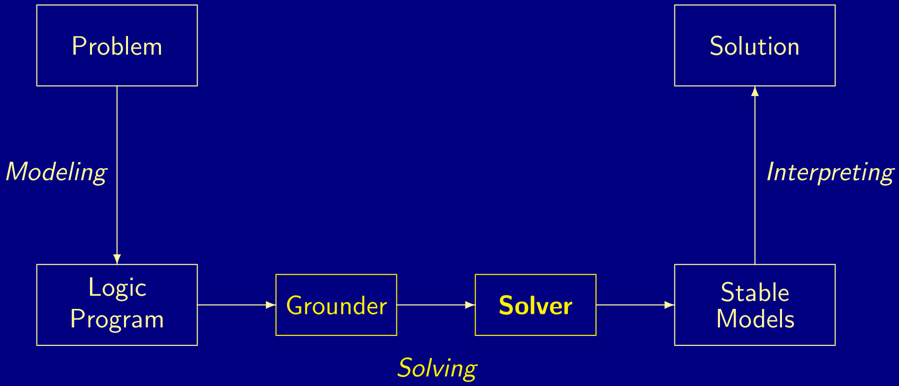

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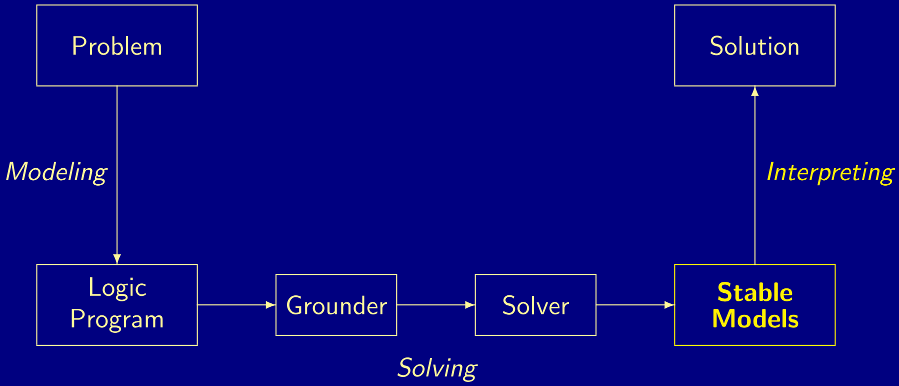

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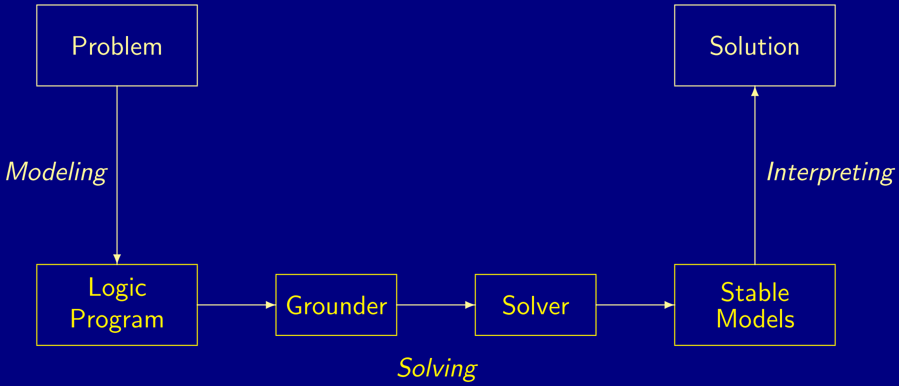

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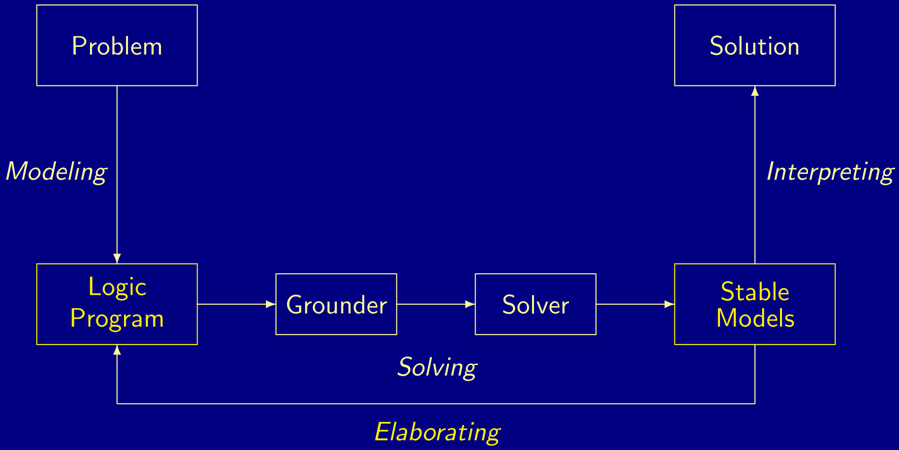
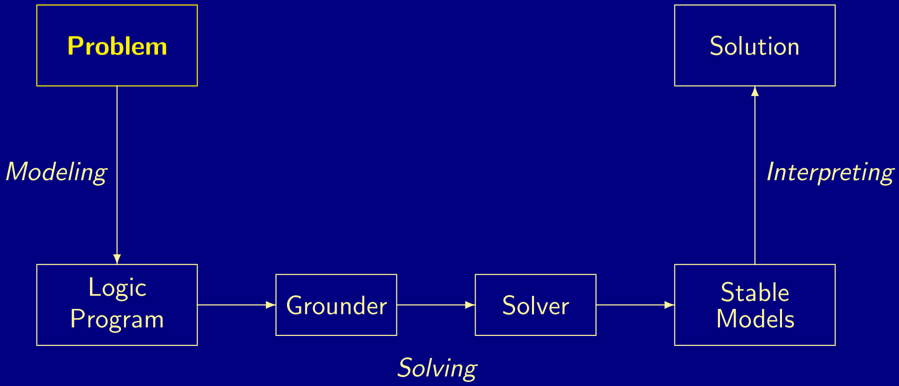


A case-study: Graph coloring



Graph coloring

- Problem instance A graph consisting of nodes and edges

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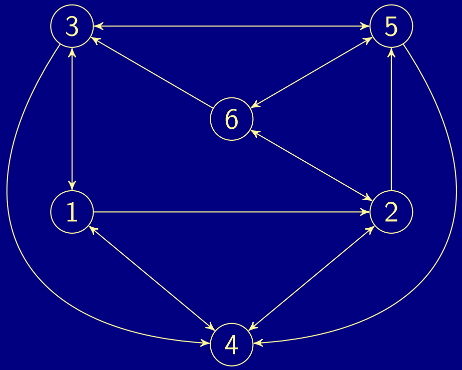

Graph coloring

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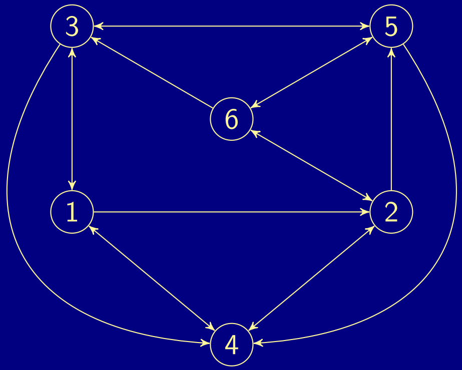

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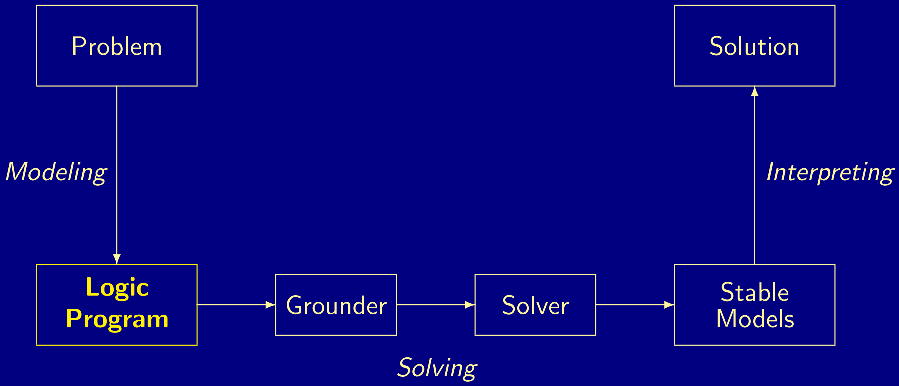
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

  In other words,

  1. Each node has a unique color
  2. Two connected nodes must not have the same color

ASP solving process



Graph coloring

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node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
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col(r).   col(b).   col(g).
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1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
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col(r).   col(b).   col(g).
```

Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

color.lp

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

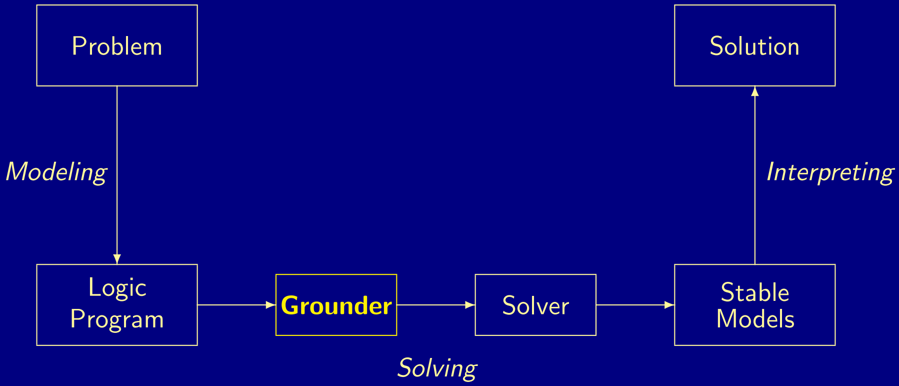
Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

ASP solving process



Graph coloring: Grounding

```
$ gringo --text color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph coloring: Grounding

```
$ gringo --text color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

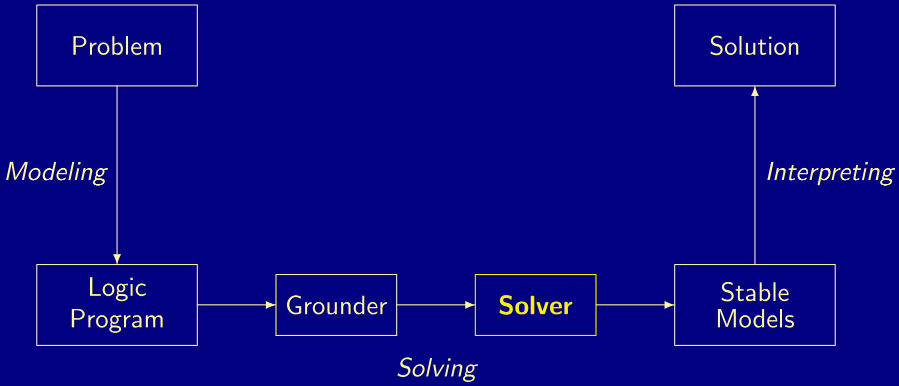
edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

ASP solving process



Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
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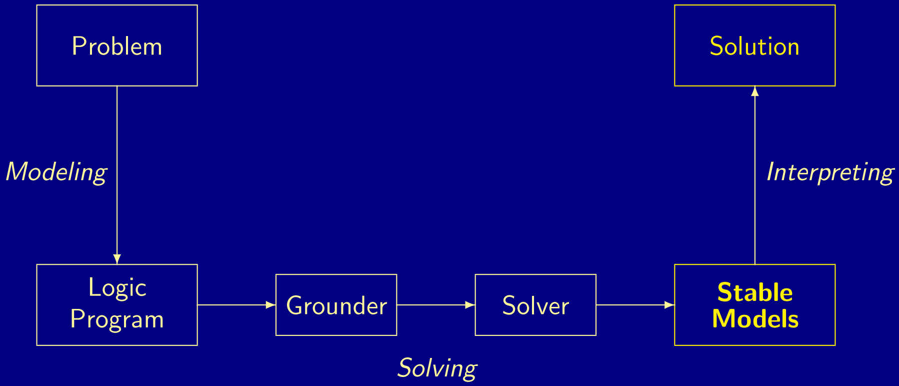
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

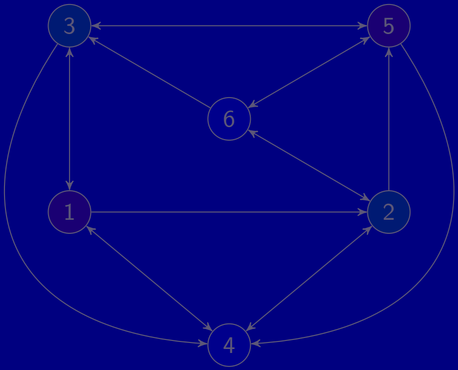
Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

ASP solving process



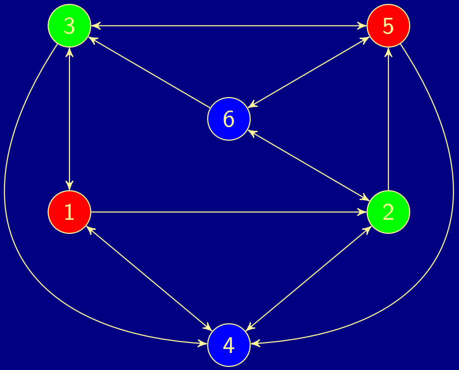
A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                      \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



Overview

13 ASP solving process

14 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- **Generator**: Generate potential stable model candidates (typically through non-deterministic constructs)
- **Tester**: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program $=$ Data $+$ Generator $+$ Tester $(+$ Optimizer$)$

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- **Generator**: Generate potential stable model candidates (typically through non-deterministic constructs)
- **Tester**: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Outline

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Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	Tester	Stable models
$\{ a, b \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow a, \sim b$	$X_2 = \{\}$

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	$\leftarrow \quad a, \sim b$	$X_2 = \{\}$

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$\{\, a, b \,\} \leftarrow$	$\leftarrow \quad {\sim}a, b$	$X_1 = \{a, b\}$
	$\leftarrow \quad a, {\sim}b$	$X_2 = \{\}$

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Generator	**Tester**	**Stable models**
$\{ a, b \} \leftarrow$	$\leftarrow \; \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow \; a, \sim b$	$X_2 = \{\}$

Outline

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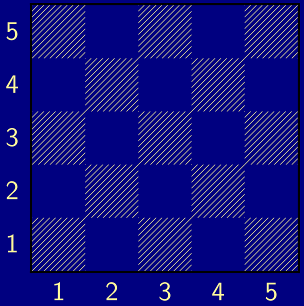
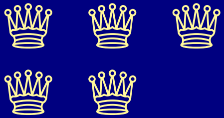
The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file queens.lp
- Define the field
  - n rows
  - n columns

Defining the Field

Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
```

- Guess a solution candidate
  by placing some queens on the board

Placing some Queens

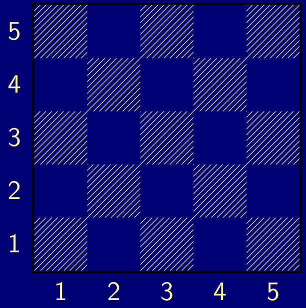
Running ...

```
$ gringo queens.lp --const n=5 | clasp 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models      : 3+
```

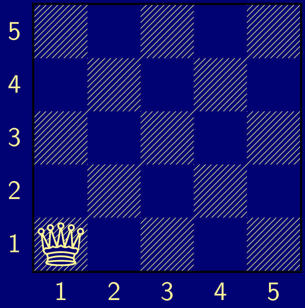
Placing some Queens: Answer 1

Answer 1



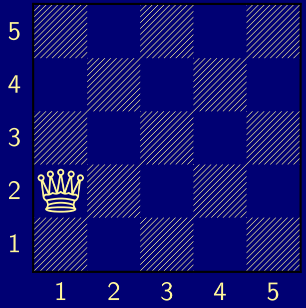
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
```

- Place exactly n queens on the board

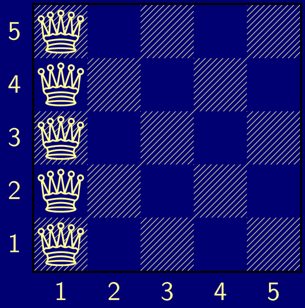
Placing n Queens

Running ...

```
$ gringo queens.lp --const n=5 | clasp 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

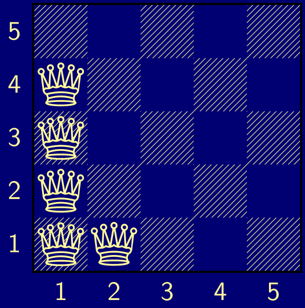
Placing n Queens: Answer 1

Answer 1



Placing *n* Queens: Answer 2

Answer 2



Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

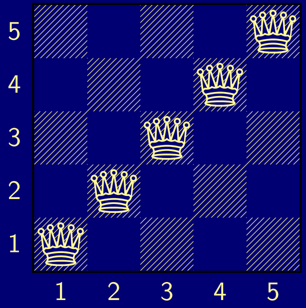
Horizontal and vertical Attack

Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I-J == II-JJ.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I+J == II+JJ.
```

- Forbid diagonal attacks

Diagonal Attack

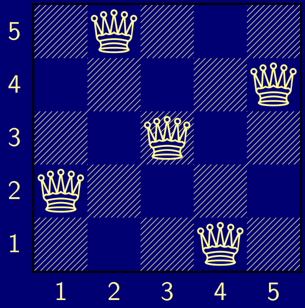
Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) \
queen(5,2) queen(2,1)
SATISFIABLE

Models      : 1+
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
1 { queen(I,1..n) } 1 :- I = 1..n.
1 { queen(1..n,J) } 1 :- J = 1..n.
 :- 2 { queen(D-J,J) }, D =   2..2*n.
 :- 2 { queen(D+J,J) }, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

Outline

13 ASP solving process

14 Methodology
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- Planning

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

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#minimize [ cycle(X,Y) = C : cost(X,Y,C) ].
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Traveling Salesperson

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Outline

13 ASP solving process

14 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Reviewer Assignment
by Ilkka Niemelä

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reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 { assigned(P,R) : reviewer(R) } 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
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Simplistic STRIPS Planning

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time(1..k).    lasttime(T) :- time(T), not time(T+1).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).       pre(a,p).      pre(b,q).
fluent(r).       add(a,q).      add(b,r).     query(r).
                 del(a,p).      del(b,q).

holds(P,0) :- init(P).

1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

ocdel(F,T) :- occ(A,T), del(A,F).
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Core Language: Overview

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17 Choice rule

18 Cardinality rule

19 Weight rule

20 Conditional literal

21 Optimization statement

22 smodels format

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Basic language extensions

- The expressiveness of a language can be enhanced by introducing new constructs
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation
- This translation might also be used for implementing the language extension

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Integrity constraint

- Idea Eliminate unwanted solution candidates
- Syntax An integrity constraint is of the form

$$\leftarrow a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n$$

  where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$
- Example `:- edge(3,7), color(3,red), color(7,red).`
- Embedding The above integrity constraint can be turned into the normal rule

$$x \leftarrow a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n, {\sim}x$$

  where x is a new symbol, that is, $x \notin \mathcal{A}$.
- Another example $P = \{a \leftarrow {\sim}b,\ b \leftarrow {\sim}a\}$
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Choice rule

- Idea Choices over subsets
- Syntax A choice rule is of the form

$$\{a_1, \ldots, a_m\} \leftarrow a_{m+1}, \ldots, a_n, {\sim}a_{n+1}, \ldots, {\sim}a_o$$

where $0 \leq m \leq n \leq o$ and each a_i is an atom for $1 \leq i \leq o$

- Informal meaning If the body is satisfied by the stable model at hand, then any subset of $\{a_1, \ldots, a_m\}$ can be included in the stable model
- Example `{ buy(pizza), buy(wine), buy(corn) } :- at(grocery).`
- Another Example $P = \{ \{a\} \leftarrow b,\ b \leftarrow \}$ has two stable models: $\{b\}$ and $\{a, b\}$

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Embedding in normal rules

- A choice rule of form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

can be translated into $2m + 1$ normal rules

$$\begin{array}{rcl} a' & \leftarrow & a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o \\ a_1 & \leftarrow & a', \sim \overline{a_1} \qquad \dots \qquad a_m \leftarrow a', \sim \overline{a_m} \\ \overline{a_1} & \leftarrow & \sim a_1 \qquad\quad \dots \qquad \overline{a_m} \leftarrow \sim a_m \end{array}$$

by introducing new atoms $a', \overline{a_1}, \dots, \overline{a_m}$.

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Cardinality rule

- Idea Control (lower) cardinality of subsets
- Syntax A cardinality rule is the form

$$a_0 \leftarrow l \; \{ \; a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \; \}$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$;
l is a non-negative integer.

- Informal meaning The head atom belongs to the stable model, if at least l elements of the body are included in the stable model
- Note l acts as a lower bound on the body
- Example `pass(c42) :- 2 { pass(a1), pass(a2), pass(a3) }.`
- Another Example $P = \{ \; a \leftarrow 1\{b, c\}, \; b \leftarrow \}$ has stable model $\{a, b\}$

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Embedding in normal rules

- Replace each cardinality rule

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \}$$

by $\quad a_0 \leftarrow ctr(1, l)$

where atom $ctr(i, j)$ represents the fact that at least j of the literals having an equal or greater index than i, are in a stable model

- The definition of $ctr/2$ is given for $0 \leq k \leq l$ by the rules

$$\begin{array}{rcll} ctr(i, k{+}1) & \leftarrow & ctr(i+1, k), a_i & \\ ctr(i, k) & \leftarrow & ctr(i+1, k) & \text{for } 1 \leq i \leq m \\ ctr(j, k{+}1) & \leftarrow & ctr(j+1, k), \sim a_j & \\ ctr(j, k) & \leftarrow & ctr(j+1, k) & \text{for } m+1 \leq j \leq n \\ ctr(n+1, 0) & \leftarrow & & \end{array}$$

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Embedding in normal rules

- Replace each cardinality rule

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \}$$

by $\quad a_0 \leftarrow ctr(1, l)$

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An example

- Program $\{\ a \leftarrow,\ \ c \leftarrow 1\ \{a, b\}\ \}$ has the stable model $\{a, c\}$
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. . . and vice versa

- A normal rule

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n,$$

can be represented by the cardinality rule

$$a_0 \leftarrow n \; \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$$

Cardinality rules with upper bounds

- A rule of the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \} \; u$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$; l and u are non-negative integers

stands for

$$\begin{array}{rcl} a_0 & \leftarrow & b, \sim c \\ b & \leftarrow & l \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \} \\ c & \leftarrow & u{+}1 \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \} \end{array}$$

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- The single constraint in the body of the above cardinality rule is referred to as a cardinality constraint

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where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$; l and u are non-negative integers

- Informal meaning A cardinality constraint is satisfied by a stable model X, if the number of its contained literals satisfied by X is between l and u (inclusive)
- In other words, if

$$l \leq |\,(\{a_1, \dots, a_m\} \cap X) \cup (\{a_{m+1}, \dots, a_n\} \setminus X)\,| \leq u$$

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Cardinality constraints as heads

- A rule of the form

$$l \ \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \ u \leftarrow a_{n+1}, \dots, a_o, \sim a_{o+1}, \dots, \sim a_p$$

where $0 \leq m \leq n \leq o \leq p$ and each a_i is an atom for $1 \leq i \leq p$;
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$$\begin{array}{rcl} b & \leftarrow & a_{n+1}, \dots, a_o, \sim a_{o+1}, \dots, \sim a_p \\ \{a_1, \dots, a_m\} & \leftarrow & b \\ c & \leftarrow & l \ \{a_1, \dots, a_m, , \sim a_{m+1}, \dots, \sim a_n\} \ u \\ & \leftarrow & b, \sim c \end{array}$$

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Overview

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Weight rule

- Syntax A weight rule is the form

$$a_0 \leftarrow l \{ a_1 = w_1, \ldots, a_m = w_m, \sim a_{m+1} = w_{m+1}, \ldots, \sim a_n = w_n \}$$

  where $0 \leq m \leq n$ and each a_i is an atom;
  l and w_i are integers for $1 \leq i \leq n$
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- Note (Cardinality and) weight constraints amount to constraints on (count and) sum aggregate functions

- Example `10 {course(db)=6,course(ai)=6,course(project)=8,course(xml)=3} 20`

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- Syntax A conditional literal is of the form

  $$\ell : \ell_1 : \cdots : \ell_n$$

  where ℓ and ℓ_i are literals for $0 \leq i \leq n$
- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \ldots, \ell_n\}$
- Note The expansion of conditional literals is context dependent
- Example Given '`p(1). p(2). p(3). q(2).`'

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r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
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- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \ldots, \ell_n\}$
- Note The expansion of conditional literals is context dependent
- Example Given '`p(1). p(2). p(3). q(2).`'

```
r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
```

  is instantiated to

```
r(1); r(3) :- r(1), r(3), 1 {r(1), r(3)}.
```

Overview

15 Motivation

16 Integrity constraint

17 Choice rule

18 Cardinality rule

19 Weight rule

20 Conditional literal

21 Optimization statement

22 smodels format

Optimization statement

- Idea Express cost functions subject to minimization and/or maximization
- Syntax A minimize statement is of the form

$$minimize\{\ \ell_1 = w_1@p_1, \dots, \ell_n = w_n@p_n\ \}.$$

where each ℓ_i is a literal; and w_i and p_i are integers for $1 \leq i \leq n$

Priority levels, p_i, allow for representing lexicographically ordered minimization objectives

- Meaning A minimize statement is a directive that instructs the ASP solver to compute optimal stable models by minimizing a weighted sum of elements

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- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

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#maximize[ hd(1)=250@1, hd(2)=500@1, hd(3)=750@1, hd(4)=1000@1 ].
#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
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The priority levels indicate that (minimizing) price is more important than (maximizing) capacity

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smodels format

- Logic programs in *smodels* format consist of
  - normal rules
  - choice rules
  - cardinality rules
  - weight rules
  - optimization statements
- Such a format is obtained by grounders *lparse* and `gringo`

Language Extensions: Overview

23 Two kinds of negation

24 Disjunctive logic programs

25 Propositional theories

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Motivation

- Classical versus default negation
  - Symbol $\neg$ and $\sim$
  - Idea
    - $\neg a \;\approx\; \neg a \in X$
    - $\sim a \;\approx\; a \notin X$
  - Example
    - $cross \leftarrow \neg train$
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Classical negation

- We consider logic programs in negation normal form
  - That is, classical negation is applied to atoms only
- Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg a \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, classical negation is encoded by adding

$$P^{\neg} = \{a \leftarrow b, \neg b \mid a \in (\mathcal{A} \cup \overline{\mathcal{A}}), b \in \mathcal{A}\}$$

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An example

- The program

$$P \;=\; \{a \leftarrow {\sim}b,\; b \leftarrow {\sim}a\} \cup \{c \leftarrow b,\; \neg c \leftarrow b\}$$

induces

$$P^{\neg} = \left\{ \begin{array}{rcl@{\qquad}rcl@{\qquad}rcl} a & \leftarrow & a,\neg a & a & \leftarrow & b,\neg b & a & \leftarrow & c,\neg c \\ \neg a & \leftarrow & a,\neg a & \neg a & \leftarrow & b,\neg b & \neg a & \leftarrow & c,\neg c \\ b & \leftarrow & a,\neg a & b & \leftarrow & b,\neg b & b & \leftarrow & c,\neg c \\ \neg b & \leftarrow & a,\neg a & \neg b & \leftarrow & b,\neg b & \neg b & \leftarrow & c,\neg c \\ c & \leftarrow & a,\neg a & c & \leftarrow & b,\neg b & c & \leftarrow & c,\neg c \\ \neg c & \leftarrow & a,\neg a & \neg c & \leftarrow & b,\neg b & \neg c & \leftarrow & c,\neg c \end{array} \right\}$$

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Properties

- The only inconsistent stable "model" is $X = \mathcal{A} \cup \overline{\mathcal{A}}$
- Note Strictly speaking, an inconsistemt set like $\mathcal{A} \cup \overline{\mathcal{A}}$ is not a model
- For a logic program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:
  1. All stable models of P are consistent or
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- $P_1 = \{cross \leftarrow {\sim}train\}$
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- $P_2 = \{cross \leftarrow \neg train\}$
  - stable model: $\emptyset$
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Default negation in rule heads

- We consider logic programs with default negation in rule heads
- Given an alphabet $\mathcal{A}$ of atoms, let $\widetilde{\mathcal{A}} = \{\widetilde{a} \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \widetilde{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, consider the program

$$\begin{aligned}
\widetilde{P} \quad = \quad & \{\, r \mid r \in P, {\sim}a \neq head(r) \,\} \\
\cup & \{\, \tilde{a} \leftarrow body(r) \mid r \in P, {\sim}a = head(r) \,\} \\
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- A set X of atoms is a stable model of a program P (with default negation in rule head) over $\mathcal{A}$,
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Disjunctive logic programs

- A disjunctive rule, r, is of the form

$$a_1 \,;\ldots\,; a_m \leftarrow a_{m+1},\ldots,a_n,\sim a_{n+1},\ldots,\sim a_o$$

  where $0 \leq m \leq n \leq o$ and each a_i is an atom for $0 \leq i \leq o$
- A disjunctive logic program is a finite set of disjunctive rules
- Notation

$$\begin{aligned} head(r) &= \{a_1,\ldots,a_m\} \\ body(r) &= \{a_{m+1},\ldots,a_n,\sim a_{n+1},\ldots,\sim a_o\} \\ body(r)^+ &= \{a_{m+1},\ldots,a_n\} \\ body(r)^- &= \{a_{n+1},\ldots,a_o\} \end{aligned}$$

- A program is called positive if $body(r)^- = \emptyset$ for all its rules

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  where $0 \leq m \leq n \leq o$ and each a_i is an atom for $0 \leq i \leq o$
- A disjunctive logic program is a finite set of disjunctive rules
- Notation

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Stable models

- Positive programs
  - A set X of atoms is closed under a positive program P iff for any $r \in P$, $head(r) \cap X \neq \emptyset$ whenever $body(r)^+ \subseteq X$
    - X corresponds to a model of P (seen as a formula)
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program P is denoted by $\min_\subseteq(P)$
    - $\min_\subseteq(P)$ corresponds to the $\subseteq$-minimal models of P (ditto)
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  - The reduct, P^X, of a disjunctive program P relative to a set X of atoms is defined by

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A "positive" example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & \\ b\,;c & \leftarrow & a \end{array} \right\}$$

- The sets $\{a, b\}$, $\{a, c\}$, and $\{a, b, c\}$ are closed under P
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Graph coloring (reloaded)

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node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
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color(X,r) | color(X,b) | color(X,g) :- node(X).


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More Examples

- $P_1 = \{a\,;b\,;c \leftarrow\}$
  - stable models $\{a\}$, $\{b\}$, and $\{c\}$
- $P_2 = \{a\,;b\,;c \leftarrow\ ,\ \leftarrow a\}$
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Some properties

- A disjunctive logic program may have zero, one, or multiple stable models
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- If X and Y are stable models of a disjunctive logic program P, then $X \not\subset Y$
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An example with variables

$$P = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(X) \,;\, c(Y) & \leftarrow & a(X,Y), \sim c(Y) \end{array} \right\}$$

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- Given a program P over $\mathcal{A}$, consider the program

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- This induces the stable models $\{a\}$ and $\emptyset$ of P

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Overview

23 Two kinds of negation

24 Disjunctive logic programs

25 Propositional theories

Propositional theories

- Formulas are formed from
  - atoms in $\mathcal{A}$
  - $\bot$

  using
  - conjunction ($\wedge$)
  - disjunction ($\vee$)
  - implication ($\rightarrow$)
- Notation

$$\begin{array}{rcl} \top & = & (\bot \rightarrow \bot) \\ \sim F & = & (F \rightarrow \bot) \end{array}$$

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Reduct

- The satisfaction relation $X \models F$ between a set X of atoms and a (set of) formula(s) F is defined as in propositional logic
- The reduct, F^X, of a formula F relative to a set X of atoms is defined recursively as follows:

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\begin{array}{ll}
F^X = \bot & \text{if } X \not\models F \\
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If $F = \sim G = (G \rightarrow \bot)$,
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Relationship to logic programs

- The translation, $\tau[(F \leftarrow G)]$, of a rule $(F \leftarrow G)$ is defined as follows:
  - $\tau[(F \leftarrow G)] = (\tau[G] \rightarrow \tau[F])$
  - $\tau[\bot] = \bot$
  - $\tau[\top] = \top$
  - $\tau[F] = F$ if F is an atom
  - $\tau[\sim F] = \sim\tau[F]$
  - $\tau[(F, G)] = (\tau[F] \land \tau[G])$
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Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$ corresponds to $\tau[P] = \{{\sim}q \rightarrow p,\ {\sim}p \rightarrow q\}$
  - stable models: $\{p\}$ and $\{q\}$
- The disjunctive logic program $P = \{p\,;q \leftarrow\}$ corresponds to $\tau[P] = \{\top \rightarrow p \vee q\}$
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Computational Aspects: Overview

26 Consequence operator

27 Computation from first principles

28 Complexity

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Consequence operator

- Let P be a positive program and X a set of atoms
  - The consequence operator T_P is defined as follows:

$$T_P X = \{ \, head(r) \mid r \in P \text{ and } body(r) \subseteq X \, \}$$

  - Iterated applications of T_P are written as T_P^j for $j \geq 0$, where
    - $T_P^0 X = X$ and
    - $T_P^i X = T_P T_P^{i-1} X$ for $i \geq 1$
- For any positive program P, we have
  - $Cn(P) = \bigcup_{i \geq 0} T_P^i \emptyset$
  - $X \subseteq Y$ implies $T_P X \subseteq T_P Y$
  - $Cn(P)$ is the smallest fixpoint of T_P

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An example

- Consider the program

$$P = \{p \leftarrow,\ q \leftarrow,\ r \leftarrow p,\ s \leftarrow q, t,\ t \leftarrow r,\ u \leftarrow v\}$$

- We get

$$\begin{array}{lclcl}
T_P^0\emptyset & = & \emptyset & & \\
T_P^1\emptyset & = & \{p, q\} & = & T_P T_P^0\emptyset & = & T_P\emptyset \\
T_P^2\emptyset & = & \{p, q, r\} & = & T_P T_P^1\emptyset & = & T_P\{p, q\} \\
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Approximating stable models

- First Idea Approximate a stable model X by two sets of atoms L and U such that $L \subseteq X \subseteq U$
  - L and U constitute lower and upper bounds on X
  - L and $(\mathcal{A} \setminus U)$ describe a three-valued model of the program

- Observation

$$X \subseteq Y \text{ implies } P^Y \subseteq P^X \text{ implies } Cn(P^Y) \subseteq Cn(P^X)$$

- Properties Let X be a stable model of normal logic program P
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Approximating stable models

- Second Idea

  repeat

  replace L **by** $L \cup Cn(P^U)$
  replace U **by** $U \cap Cn(P^L)$

  until L *and* U *do not change anymore*

- Observations
  - At each iteration step
    - L becomes larger (or equal)
    - U becomes smaller (or equal)
  - $L \subseteq X \subseteq U$ is invariant for every stable model X of P

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The simplistic expand algorithm

expand$_P(L, U)$

repeat

$L' \leftarrow L$

$U' \leftarrow U$

$L \leftarrow L' \cup Cn(P^{U'})$

$U \leftarrow U' \cap Cn(P^{L'})$

if $L \not\subseteq U$ **then return**

until $L = L'$ and $U = U'$

An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\emptyset$	$\{a\}$	$\{a\}$	$\{a,b,c,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
2	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
3	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$

- Note We have $\{a,b\} \subseteq X$ and $(\mathcal{A} \setminus \{a,b,d,e\}) \cap X = (\{c\} \cap X) = \emptyset$ for every stable model X of P

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The simplistic expand algorithm

- **expand**$_P$
  - tightens the approximation on stable models
  - is stable model preserving

Let's expand with d !

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\{d\}$	$\{a\}$	$\{a, d\}$	$\{a, b, c, d, e\}$	$\{a, b, d\}$	$\{a, b, d\}$
2	$\{a, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$
3	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$

- Note $\{a, b, d\}$ is a stable model of P

Let's expand with d !

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	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\{d\}$	$\{a\}$	$\{a, d\}$	$\{a, b, c, d, e\}$	$\{a, b, d\}$	$\{a, b, d\}$
2	$\{a, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$
3	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$

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A simplistic solving algorithm

$solve_P(L, U)$

$(L, U) \leftarrow expand_P(L, U)$ // *propagation*

if $L \not\subseteq U$ **then failure** // *failure*

if $L = U$ **then output** L // *success*

else choose $a \in U \setminus L$ // *choice*

$solve_P(L \cup \{a\}, U)$

$solve_P(L, U \setminus \{a\})$

A simplistic solving algorithm

- Close to the approach taken by the ASP solver `smodels`, inspired by the Davis-Putman-Logemann-Loveland (DPLL) procedure
  - Backtracking search building a binary search tree
  - A node in the search tree corresponds to a three-valued interpretation
  - The search space is pruned by
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    - making one choice at a time by appeal to a heuristic (**choose**)
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Overview

26 Consequence operator

27 Computation from first principles

28 Complexity

Complexity

Let a be an atom and X be a set of atoms

- For a positive normal logic program P:
  - Deciding whether X is the stable model of P is P-complete
  - Deciding whether a is in the stable model of P is P-complete
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  - Deciding whether X is a stable model of P is P-complete
  - Deciding whether a is in a stable model of P is NP-complete
- For a normal logic program P with optimization statements:
  - Deciding whether X is an optimal stable model of P is $co\text{-}NP$-complete
  - Deciding whether a is in an optimal stable model of P is Δ_2^p-complete

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Axiomatic Characterization: Overview

29 Completion

30 Tightness

31 Loops and Loop Formulas

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Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?

- Observation Although each atom is defined through a set of rules, each such rule provides only a sufficient condition for its head atom

- Idea The idea of program completion is to turn such implications into a definition by adding the corresponding necessary counterpart

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Program completion

Let P be a normal logic program

- The completion $CF(P)$ of P is defined as follows

$$CF(P) = \left\{ a \leftrightarrow \bigvee_{r \in P, head(r)=a} BF(body(r)) \mid a \in atom(P) \right\}$$

where

$$BF(body(r)) = \bigwedge_{a \in body(r)^+} a \wedge \bigwedge_{a \in body(r)^-} \neg a$$

An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow \sim a \\ c \leftarrow a, \sim d \\ d \leftarrow \sim c, \sim e \\ e \leftarrow b, \sim f \\ e \leftarrow e \end{array} \right\} \qquad CF(P) = \left\{ \begin{array}{l} a \leftrightarrow \top \\ b \leftrightarrow \neg a \\ c \leftrightarrow a \wedge \neg d \\ d \leftrightarrow \neg c \wedge \neg e \\ e \leftrightarrow (b \wedge \neg f) \vee e \\ f \leftrightarrow \bot \end{array} \right\}$$

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A closer look

- $CF(P)$ is logically equivalent to $\overleftarrow{CF}(P) \cup \overrightarrow{CF}(P)$, where

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Supported models

- Every stable model of P is a model of $CF(P)$, but not vice versa

  Models of $CF(P)$ are called the supported models of P

  In other words, every stable model of P is a supported model of P

  By definition, every supported model of P is also a model of P

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An example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- P has 21 models, including $\{a, c\}$, $\{a, d\}$, but also $\{a, b, c, d, e, f\}$
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Overview

29 Completion

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The mismatch

- Question What causes the mismatch between supported models and stable models?
- Hint Consider the unstable yet supported model $\{a, c, e\}$ of P !
- Answer Cyclic derivations are causing the mismatch between supported and stable models
  - Atoms in a stable model can be "derived" from a program in a finite number of steps
  - Atoms in a cycle (not being "supported from outside the cycle") cannot be "derived" from a program in a finite number of steps
    - But such atoms do not contradict the completion of a program and do thus not eliminate an unstable supported model

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Non-cyclic derivations

Let X be a stable model of normal logic program P

- For every atom $A \in X$, there is a finite sequence of positive rules

  $$\langle r_1, \dots, r_n \rangle$$

  such that

  1. $head(r_1) = A$
  2. $body(r_i)^+ \subseteq \{head(r_j) \mid i < j \leq n\}$ for $1 \leq i \leq n$
  3. $r_i \in P^X$ for $1 \leq i \leq n$

- That is, each atom of X has a non-cyclic derivation from P^X

- Example There is no finite sequence of rules providing a derivation for e from $P^{\{a,c,e\}}$

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- The origin of (potential) circular derivations can be read off the positive atom dependency graph $G(P)$ of a logic program P given by

$$(atom(P), \{(a, b) \mid r \in P, a \in body(r)^+, head(r) = b\})$$

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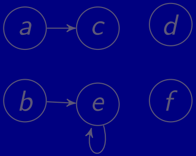
- $G(P) = (\{a, b, c, d, e\}, \{(a, c), (b, e), (e, e)\})$



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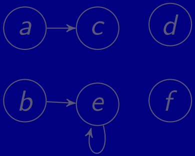

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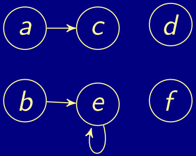

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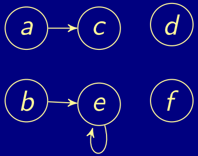

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- For tight programs, stable and supported models coincide:

Fages' Theorem

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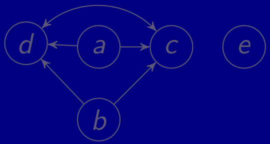

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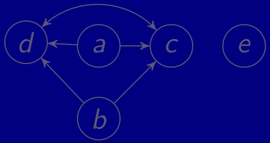

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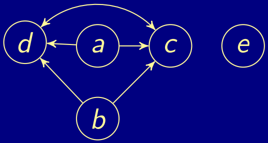

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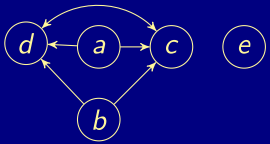

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Overview

29 Completion

30 Tightness

31 Loops and Loop Formulas

Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?

- Observation Starting from the completion of a program, the problem boils down to eliminating the circular support of atoms holding in the supported models of the program

- Idea Add formulas prohibiting circular support of sets of atoms

- Note Circular support between atoms a and b is possible, if a has a path to b and b has a path to a in the program's positive atom dependency graph

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Let P be a normal logic program, and
let $G(P) = (atom(P), E)$ be the positive atom dependency graph of P

- A set $\emptyset \subset L \subseteq atom(P)$ is a loop of P
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  That is, each pair of atoms in L is connected by a path of non-zero length in $(L, E \cap (L \times L))$

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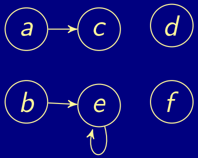

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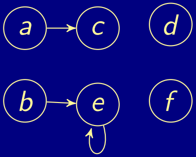

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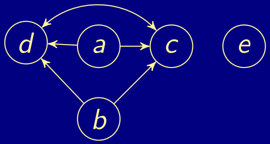

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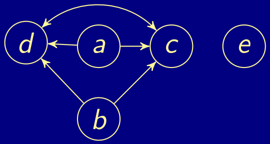

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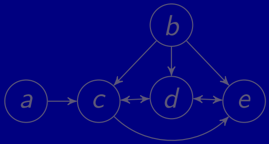

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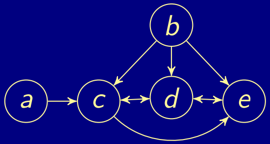

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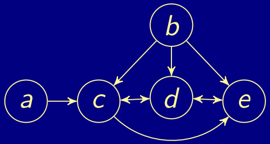

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Loop formulas

Let P be a normal logic program

- For $L \subseteq atom(P)$, define the external supports of L for P as

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- Define the external bodies of L in P as $EB_P(L) = body(ES_P(L))$
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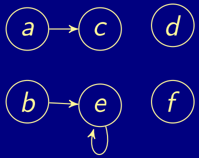

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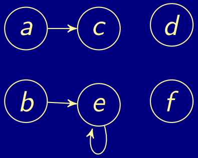

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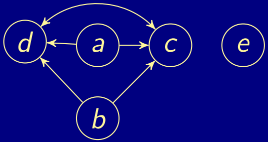

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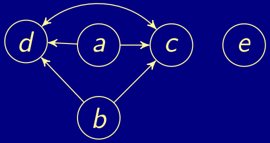

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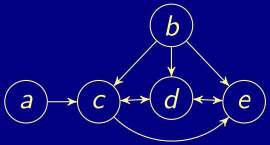

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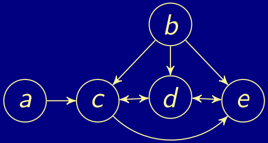

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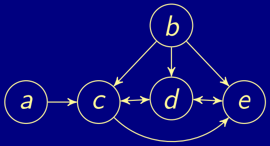

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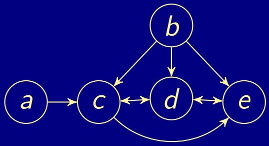

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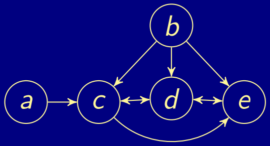

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- $LF(P) = \left\{ \begin{array}{l} c \vee d \rightarrow a \vee e \\ d \vee e \rightarrow (b \wedge c) \vee (b \wedge \neg a) \\ c \vee d \vee e \rightarrow a \vee b \end{array} \right\}$

Lin-Zhao Theorem

Theorem

Let P be a normal logic program and $X \subseteq atom(P)$
Then, X is a stable model of P iff $X \models CF(P) \cup LF(P)$

Loops and loop formulas: Properties

Let X be a supported model of normal logic program P

- Then, X is a stable model of P iff
  - $X \models \{ LF_P(U) \mid U \subseteq atom(P) \}$;
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  - $X \models \{ LF_P(L) \mid L \in loop(P), L \subseteq X \}$
- Note If X is not a stable model of P,
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Loops and loop formulas: Properties (ctd)

If $\mathcal{P} \not\subseteq \mathcal{NC}^1/\mathit{poly}$,[1] then there is no translation $\mathcal{T}$ from logic programs to propositional formulas such that, for each normal logic program P, both of the following conditions hold:

1. The propositional variables in $\mathcal{T}[P]$ are a subset of $atom(P)$
2. The size of $\mathcal{T}[P]$ is polynomial in the size of P
   - Note Every vocabulary-preserving translation from normal logic programs to propositional formulas must be exponential (in the worst case).

Observations

- Translation $CF(P) \cup LF(P)$ preserves the vocabulary of P
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[1] A conjecture from the theory of complexity that is widely believed to be true.

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Operational Characterization: Overview

32 Partial Interpretations

33 Fitting Operator

34 Unfounded Sets

35 Well-Founded Operator

Overview

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Interlude: Partial interpretations

or: 3-valued interpretations

A partial interpretation maps atoms onto truth values *true*, *false*, and *unknown*

- Representation $\langle T, F \rangle$, where
  - T is the set of all *true* atoms and
  - F is the set of all *false* atoms
  - Truth of atoms in $\mathcal{A} \setminus (T \cup F)$ is *unknown*
- Properties
  - $\langle T, F \rangle$ is conflicting if $T \cap F \neq \emptyset$
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Overview

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Basic idea

- Idea Extend T_P to normal logic programs
- Logical background The idea is to turn a program's completion into an operator such that
  - the head atom of a rule must be *true*
    if the rule's body is *true*
  - an atom must be *false*
    if the body of each rule having it as head is *false*

Definition

- Let P be a normal logic program
- Define

$$\mathbf{\Phi}_P\langle T,F\rangle = \langle \mathbf{T}_P\langle T,F\rangle, \mathbf{F}_P\langle T,F\rangle\rangle$$

where

$$\mathbf{T}_P\langle T,F\rangle = \{\ \mathit{head}(r) \mid r \in P, \mathit{body}(r)^+ \subseteq T, \mathit{body}(r)^- \subseteq F\ \}$$

$$\begin{aligned}\mathbf{F}_P\langle T,F\rangle = \{\ a \in \mathit{atom}(P) \mid \\ \mathit{body}(r)^+ \cap F \neq \emptyset \text{ or } \mathit{body}(r)^- \cap T \neq \emptyset \\ \text{for each } r \in P \text{ such that } \mathit{head}(r) = a\ \}\end{aligned}$$

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Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Phi}_P$ on $\langle \{a\}, \{d\} \rangle$:

$$\begin{array}{rcl} \mathbf{\Phi}_P \langle \{a\}, \{d\} \rangle & = & \langle \{a, c\}, \{b, f\} \rangle \\ \mathbf{\Phi}_P \langle \{a, c\}, \{b, f\} \rangle & = & \langle \{a\}, \{b, d, f\} \rangle \\ \mathbf{\Phi}_P \langle \{a\}, \{b, d, f\} \rangle & = & \langle \{a, c\}, \{b, f\} \rangle \\ & \vdots & \end{array}$$

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Fitting semantics

- Define the iterative variant of $\mathbf{\Phi}_P$ analogously to T_P:

$$\mathbf{\Phi}^0_P\langle T,F\rangle = \langle T,F\rangle \qquad \mathbf{\Phi}^{i+1}_P\langle T,F\rangle = \mathbf{\Phi}_P\mathbf{\Phi}^i_P\langle T,F\rangle$$

- Define the Fitting semantics of a normal logic program P as the partial interpretation:

$$\bigsqcup_{i\geq 0}\mathbf{\Phi}^i_P\langle\emptyset,\emptyset\rangle$$

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Properties

Let P be a normal logic program

- $\mathbf{\Phi}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Phi}_P^i\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Phi}_P^{i+1}\langle\emptyset,\emptyset\rangle$
- The Fitting semantics of P is
  - not conflicting,
  - and generally not total

Fitting fixpoints

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Define $\langle T, F\rangle$ as a Fitting fixpoint of P if $\mathbf{\Phi}_P\langle T, F\rangle = \langle T, F\rangle$
  - The Fitting semantics is the $\sqsubseteq$-least Fitting fixpoint of P
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  - Total Fitting fixpoints correspond to supported models

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$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, {\sim}d & e \leftarrow b, {\sim}f \\ b \leftarrow {\sim}a & d \leftarrow {\sim}c, {\sim}e & e \leftarrow e \end{array} \right\}$$

- P has three total Fitting fixpoints:
  - $\langle\{a, c\}, \{b, d, e\}\rangle$
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Let P be a normal logic program,
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- Let $\mathbf{\Phi}_P\langle T, F\rangle = \langle T', F'\rangle$
- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Phi}_P$ is stable model preserving

  Hence, $\mathbf{\Phi}_P$ can be used for approximating stable models and so for propagation in ASP-solvers

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Unfounded sets

Let P be a normal logic program,
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- A set $U \subseteq atom(P)$ is an unfounded set of P wrt $\langle T, F\rangle$, if we have for each rule $r \in P$ such that $head(r) \in U$ either
  1. $body(r)^+ \cap F \neq \emptyset$ or $body(r)^- \cap T \neq \emptyset$ or
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- Intuitively, $\langle T, F\rangle$ is what we already know about P
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- Rules satisfying Condition 1 are not usable for further derivations
- Condition 3 is the unfounded set condition treating cyclic derivations: All rules still being usable to derive an atom in U require an(other) atom in U to be true

Unfounded sets

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- A set $U \subseteq \mathit{atom}(P)$ is an unfounded set of P wrt $\langle T, F\rangle$, if we have for each rule $r \in P$ such that $\mathit{head}(r) \in U$ either
  1. $\mathit{body}(r)^+ \cap F \neq \emptyset$ or $\mathit{body}(r)^- \cap T \neq \emptyset$ or $\mathit{body}(r)^+ \cap U \neq \emptyset$
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Example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & b \\ b & \leftarrow & a \end{array} \right\}$$

- $\emptyset$ is an unfounded set (by definition)
- $\{a\}$ is not an unfounded set of P wrt $\langle\emptyset, \emptyset\rangle$
- $\{a\}$ is an unfounded set of P wrt $\langle\emptyset, \{b\}\rangle$
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- Analogously for $\{b\}$
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Greatest unfounded sets

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Observation The union of two unfounded sets is an unfounded set
- The greatest unfounded set of P wrt $\langle T, F\rangle$ is the union of all unfounded sets of P wrt $\langle T, F\rangle$

  It is denoted by $\mathbf{U}_P\langle T, F\rangle$
- Alternatively, we may define

$$\mathbf{U}_P\langle T, F\rangle = atom(P) \setminus Cn(\{r \in P \mid body(r)^+ \cap F = \emptyset\}^T)$$

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Overview

32 Partial Interpretations

33 Fitting Operator

34 Unfounded Sets

35 Well-Founded Operator

Well-founded operator

Let P be a normal logic program,
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- Observation Condition 1 (in the definition of an unfounded set) corresponds to $\mathbf{F}_P\langle T, F\rangle$ of Fitting's $\mathbf{\Phi}_P\langle T, F\rangle$
- Idea Extend (negative part of) Fitting's operator $\mathbf{\Phi}_P$
  That is,
  - keep definition of $\mathbf{T}_P\langle T, F\rangle$ from $\mathbf{\Phi}_P\langle T, F\rangle$ and
  - replace $\mathbf{F}_P\langle T, F\rangle$ from $\mathbf{\Phi}_P\langle T, F\rangle$ by $\mathbf{U}_P\langle T, F\rangle$
- In words, an atom must be *false*
  if it belongs to the greatest unfounded set
- Definition $\mathbf{\Omega}_P\langle T, F\rangle = \langle \mathbf{T}_P\langle T, F\rangle, \mathbf{U}_P\langle T, F\rangle\rangle$
  $\mathbf{\Phi}_P\langle T, F\rangle \sqsubseteq \mathbf{\Omega}_P\langle T, F\rangle$

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Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Omega}_{P_1}$ on $\langle \{c\}, \emptyset \rangle$:

$$\begin{array}{rcl} \mathbf{\Omega}_P \langle \{c\}, \emptyset \rangle & = & \langle \{a\}, \{d, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a\}, \{d, f\} \rangle & = & \langle \{a, c\}, \{b, e, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a, c\}, \{b, e, f\} \rangle & = & \langle \{a\}, \{b, d, e, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a\}, \{b, d, e, f\} \rangle & = & \langle \{a, c\}, \{b, e, f\} \rangle \\ & \vdots & \end{array}$$

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Well-founded semantics

- Define the iterative variant of $\mathbf{\Omega}_P$ analogously to $\mathbf{\Phi}_P$:

$$\mathbf{\Omega}_P^0\langle T, F\rangle = \langle T, F\rangle \qquad \mathbf{\Omega}_P^{i+1}\langle T, F\rangle = \mathbf{\Omega}_P\mathbf{\Omega}_P^i\langle T, F\rangle$$

- Define the well-founded semantics of a normal logic program P as the partial interpretation:

$$\bigsqcup_{i\geq 0}\mathbf{\Omega}_P^i\langle\emptyset, \emptyset\rangle$$

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Properties

Let P be a normal logic program

- $\mathbf{\Omega}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Omega}^i_P\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Omega}^{i+1}_P\langle\emptyset,\emptyset\rangle$
- The well-founded semantics of P is
  - not conflicting,
  - and generally not total
- We have $\bigsqcup_{i\geq 0} \mathbf{\Phi}^i_P\langle\emptyset,\emptyset\rangle \sqsubseteq \bigsqcup_{i\geq 0} \mathbf{\Omega}^i_P\langle\emptyset,\emptyset\rangle$

Well-founded fixpoints

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Define $\langle T, F\rangle$ as a well-founded fixpoint of P if $\mathbf{\Omega}_P\langle T, F\rangle = \langle T, F\rangle$
  - The well-founded semantics is the $\sqsubseteq$-least well-founded fixpoint of P
  - Any other well-founded fixpoint extends the well-founded semantics
  - Total well-founded fixpoints correspond to stable models

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- P has two total well-founded fixpoints:
  1. $\langle \{a, c\}, \{b, d, e\} \rangle$
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Properties

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Let $\mathbf{\Omega}_P\langle T, F\rangle = \langle T', F'\rangle$
- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Omega}_P$ is stable model preserving

  Hence, $\mathbf{\Omega}_P$ can be used for approximating stable models and so for propagation in ASP-solvers
- In contrast to $\mathbf{\Phi}_P$, operator $\mathbf{\Omega}_P$ is sufficient for propagation because total fixpoints correspond to stable models
- Note In addition to $\mathbf{\Omega}_P$, most ASP-solvers apply backward propagation, originating from program completion
(although this is unnecessary from a formal point of view)

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- In contrast to $\mathbf{\Phi}_P$, operator $\mathbf{\Omega}_P$ is sufficient for propagation because total fixpoints correspond to stable models

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Proof-theoretic Characterization: Overview

Motivation

- Goal Analyze computations in ASP solvers
- Wanted A declarative and fine-grained instrument for characterizing operations as well as strategies of ASP solvers
- Idea View stable model computations as derivations in an inference system

  Consider Tableau-based proof systems for analyzing ASP solving

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Tableau calculi

- Traditionally, tableau calculi are used for
  - automated theorem proving and
  - proof theoretical analysis

  in classical as well as non-classical logics
- General idea Given an input, prove some property by decomposition
  Decomposition is done by applying deduction rules
- For details, see *Handbook of Tableau Methods*, Kluwer, 1999

General definitions

- A tableau is a (mostly binary) tree
- A branch in a tableau is a path from the root to a leaf
- A branch containing $\gamma_1, \dots, \gamma_m$ can be extended by applying tableau rules of form

$$\frac{\gamma_1, \dots, \gamma_m}{\begin{array}{c}\alpha_1\\ \vdots\\ \alpha_n\end{array}} \qquad\qquad \frac{\gamma_1, \dots, \gamma_m}{\beta_1 \mid \dots \mid \beta_n}$$

- Rules of the former format append entries $\alpha_1, \dots, \alpha_n$ to the branch
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Example

- A simple tableau calculus for proving unsatisfiability of propositional formulas, composed from $\neg$, $\wedge$, and $\vee$, consists of rules

$$\frac{\neg\neg\alpha}{\alpha} \qquad \frac{\alpha_1 \wedge \alpha_2}{\begin{array}{c}\alpha_1\\ \alpha_2\end{array}} \qquad \frac{\beta_1 \vee \beta_2}{\beta_1 \mid \beta_2}$$

- All rules are semantically valid, when interpreting entries in a branch conjunctively and distinct (sub-)branches as connected disjunctively
- A propositional formula φ is unsatisfiable iff there is a tableau with φ as the root node such that
  1. all other entries can be produced by tableau rules and
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(2)	a	[1]
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All three branches of the tableau are contradictory (cf 2, 5, 7, 8, 10)

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ASP-specific definitions

- A (signed) tableau for a logic program P is a binary tree such that
  - the root node of the tree consists of the rules in P;
  - the other nodes in the tree are entries of the form $\mathbf{T}v$ or $\mathbf{F}v$, called signed literals, where v is a variable,
  - generated by extending a tableau using deduction rules (given below)
- An entry $\mathbf{T}v$ ($\mathbf{F}v$) reflects that variable v is *true* (*false*) in a corresponding variable assignment

  A set of signed literals constitutes a partial assignment
- For a normal logic program P,
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Tableau rules for ASP at a glance

$$(\mathit{FTB})\quad \frac{p \leftarrow l_1, \dots, l_n \qquad \mathbf{t}l_1, \dots, \mathbf{t}l_n}{\mathbf{T}\{l_1, \dots, l_n\}} \qquad\qquad (\mathit{BFB})\quad \frac{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\} \qquad \mathbf{t}l_1, \dots, \mathbf{t}l_{i-1}, \mathbf{t}l_{i+1}, \dots, \mathbf{t}l_n}{\mathbf{f}l_i}$$

$$(\mathit{FTA})\quad \frac{p \leftarrow l_1, \dots, l_n \qquad \mathbf{T}\{l_1, \dots, l_n\}}{\mathbf{T}p} \qquad\qquad (\mathit{BFA})\quad \frac{p \leftarrow l_1, \dots, l_n \qquad \mathbf{F}p}{\mathbf{F}\{l_1, \dots, l_n\}}$$

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$$(\mathit{FFA})\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\ (\S) \qquad\qquad (\mathit{BTA})\quad \frac{\mathbf{T}p \qquad \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m}{\mathbf{T}B_i}$$

$$(\mathit{WFN})\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\ (\dagger) \qquad\qquad (\mathit{WFJ})\quad \frac{\mathbf{T}p \qquad \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m}{\mathbf{T}B_i}$$

$$(\mathit{FL})\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\ (\ddagger) \qquad\qquad (\mathit{BL})\quad \frac{\mathbf{T}p \qquad \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m}{\mathbf{T}B_i}$$

$$(\mathit{Cut}[X])\quad \frac{}{\mathbf{T}v \mid \mathbf{F}v}\ (\sharp[X])$$

More concepts

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- A branch in a tableau is total for a program P,
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- A branch in a tableau of some calculus $\mathcal{T}$ is closed,
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  if no rule in $\mathcal{T}$ other than *Cut* can produce any new entries
- A branch in a tableau is complete,
  if it is either conflicting or both total and closed
- A tableau is complete, if all its branches are complete
- A tableau of some calculus $\mathcal{T}$ is a refutation of $\mathcal{T}$ for a program P,
  if every branch in the tableau is conflicting

Example

- Consider the program

$$P = \left\{ \begin{array}{l} a \leftarrow \\ c \leftarrow \sim b, \sim d \\ d \leftarrow a, \sim c \end{array} \right\}$$

having stable models $\{a, c\}$ and $\{a, d\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow {\sim}b, {\sim}d$$
$$d \leftarrow a, {\sim}c$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{{\sim}b, {\sim}d\}$	(BFA)	$\mathbf{F}\{{\sim}b, {\sim}d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, {\sim}c\}$	(FTB)	$\mathbf{T}\{a, {\sim}c\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
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(FTB)		$\mathbf{T}\emptyset$		
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(BTA)	$\mathbf{T}\{\sim b, \sim d\}$		(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
(BTB)	$\mathbf{F}d$		(BFB)	$\mathbf{T}d$
(FFB)	$\mathbf{F}\{a, \sim c\}$		(FTB)	$\mathbf{T}\{a, \sim c\}$

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	(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

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(FTA)		$\mathbf{T}a$		
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(Cut[$atom(P)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{\sim b, \sim d\}$	(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])	$\mathbf{T}c$			$\mathbf{F}c$
(BTA)	$\mathbf{T}\{\sim b, \sim d\}$		(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
(BTB)	$\mathbf{F}d$		(BFB)	$\mathbf{T}d$
(FFB)	$\mathbf{F}\{a, \sim c\}$		(FTB)	$\mathbf{T}\{a, \sim c\}$

(Previewed) Example

$$
\begin{array}{c}
a \leftarrow \\
c \leftarrow \sim b, \sim d \\
d \leftarrow a, \sim c
\end{array}
$$

$$
\begin{array}{rcccc}
\text{(FTB)} & & & \mathbf{T}\emptyset & & \\
\text{(FTA)} & & & \mathbf{T}a & & \\
\text{(FFA)} & & & \mathbf{F}b & & \\
(\text{Cut}[atom(P)]) & & \mathbf{T}c & & & \mathbf{F}c \\
& \text{(BTA)} & \mathbf{T}\{\sim b, \sim d\} & & \text{(BFA)} & \mathbf{F}\{\sim b, \sim d\} \\
& \text{(BTB)} & \mathbf{F}d & & \text{(BFB)} & \mathbf{T}d \\
& \text{(FFB)} & \mathbf{F}\{a, \sim c\} & & \text{(FTB)} & \mathbf{T}\{a, \sim c\}
\end{array}
$$

Auxiliary definitions

- For a literal l, define conjugation functions $\mathbf{t}$ and $\mathbf{f}$ as follows

$$\mathbf{t}l = \begin{cases} \mathbf{T}l & \text{if } l \text{ is an atom} \\ \mathbf{F}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

$$\mathbf{f}l = \begin{cases} \mathbf{F}l & \text{if } l \text{ is an atom} \\ \mathbf{T}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

- Examples $\mathbf{t}a = \mathbf{T}a$, $\mathbf{f}a = \mathbf{F}a$, $\mathbf{t}{\sim}a = \mathbf{F}a$, and $\mathbf{f}{\sim}a = \mathbf{T}a$

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Auxiliary definitions

- Some tableau rules require conditions for their application
- Such conditions are specified as provisos

$$\frac{\mathit{prerequisites}}{\mathit{consequence}}\ (\mathit{proviso})$$

proviso: some condition(s)

- Note All tableau rules given in the sequel are stable model preserving

Forward True Body (FTB)

- Prerequisites All of a body's literals are *true*
- Consequence The body is *true*
- Tableau Rule FTB

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{t}l_1, \dots, \mathbf{t}l_n \end{array}}{\mathbf{T}\{l_1, \dots, l_n\}}$$

- Example

$$\frac{\begin{array}{c} a \leftarrow b, {\sim}c \\ \mathbf{T}b \\ \mathbf{F}c \end{array}}{\mathbf{T}\{b, {\sim}c\}}$$

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- Example

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}b \\ \mathbf{F}c \end{array}}{\mathbf{T}\{b, \sim c\}}$$

Backward False Body (BFB)

- Prerequisites A body is *false*, and all its literals except for one are *true*
- Consequence The residual body literal is *false*
- Tableau Rule BFB

$$\frac{\begin{array}{c}\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\}\\ \mathbf{t}l_1,\dots,\mathbf{t}l_{i-1},\mathbf{t}l_{i+1},\dots,\mathbf{t}l_n\end{array}}{\mathbf{f}l_i}$$

- Examples

$$\frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{T}b\end{array}}{\mathbf{T}c} \qquad \frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{F}c\end{array}}{\mathbf{F}b}$$

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- Examples

$$\frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{T}b\end{array}}{\mathbf{T}c} \qquad \frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{F}c\end{array}}{\mathbf{F}b}$$

Forward False Body (FFB)

- Prerequisites Some literal of a body is *false*
- Consequence The body is *false*
- Tableau Rule FFB

$$\frac{\begin{array}{c} p \leftarrow l_1, \ldots, l_i, \ldots, l_n \\ \mathbf{f}\, l_i \end{array}}{\mathbf{F}\{l_1, \ldots, l_i, \ldots, l_n\}}$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{F}b \end{array}}{\mathbf{F}\{b, \sim c\}} \qquad \frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}c \end{array}}{\mathbf{F}\{b, \sim c\}}$$

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- Examples

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{F}\, b \end{array}}{\mathbf{F}\{b, \sim c\}} \qquad \frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}\, c \end{array}}{\mathbf{F}\{b, \sim c\}}$$

Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
- Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1,\dots,l_i,\dots,l_n\}}{\mathbf{t}l_i}$$

- Examples

$$\frac{\mathbf{T}\{b,\sim c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b,\sim c\}}{\mathbf{F}c}$$

Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
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- Examples

$$\frac{\mathbf{T}\{b,\sim c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b,\sim c\}}{\mathbf{F}c}$$

Tableau rules for bodies

Consider rule body $B = \{l_1, \dots, l_n\}$

- Rules FTB and BFB amount to implication

$$l_1 \wedge \cdots \wedge l_n \rightarrow B$$

- Rules FFB and BTB amount to implication

$$B \rightarrow l_1 \wedge \cdots \wedge l_n$$

- Together they yield

$$B \equiv l_1 \wedge \cdots \wedge l_n$$

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- Together they yield

  $$B \equiv l_1 \wedge \cdots \wedge l_n$$

Forward True Atom (FTA)

- Prerequisites Some of an atom's bodies is *true*
- Consequence The atom is *true*
- Tableau Rule FTA

$$\frac{p \leftarrow l_1, \ldots, l_n \quad \mathbf{T}\{l_1, \ldots, l_n\}}{\mathbf{T}p}$$

- Examples

$$\frac{a \leftarrow b, \sim c \quad \mathbf{T}\{b, \sim c\}}{\mathbf{T}a} \qquad \frac{a \leftarrow d, \sim e \quad \mathbf{T}\{d, \sim e\}}{\mathbf{T}a}$$

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- Prerequisites Some of an atom's bodies is *true*
- Consequence The atom is *true*
- Tableau Rule FTA

$$\frac{p \leftarrow l_1, \dots, l_n \qquad \mathbf{T}\{l_1, \dots, l_n\}}{\mathbf{T}p}$$

- Examples

$$\frac{a \leftarrow b, \sim c \qquad \mathbf{T}\{b, \sim c\}}{\mathbf{T}a} \qquad \frac{a \leftarrow d, \sim e \qquad \mathbf{T}\{d, \sim e\}}{\mathbf{T}a}$$

Backward False Atom (BFA)

- Prerequisites An atom is *false*
- Consequence The bodies of all rules with the atom as head are *false*
- Tableau Rule BFA

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{F}p \end{array}}{\mathbf{F}\{l_1, \dots, l_n\}}$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{F}a \end{array}}{\mathbf{F}\{b, \sim c\}} \qquad \frac{\begin{array}{c} a \leftarrow d, \sim e \\ \mathbf{F}a \end{array}}{\mathbf{F}\{d, \sim e\}}$$

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$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{F}a \end{array}}{\mathbf{F}\{b, \sim c\}} \qquad \frac{\begin{array}{c} a \leftarrow d, \sim e \\ \mathbf{F}a \end{array}}{\mathbf{F}\{d, \sim e\}}$$

Forward False Atom (FFA)

- Prerequisites For some atom, the bodies of all rules with the atom as head are *false*
- Consequence The atom is *false*
- Tableau Rule FFA

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \;(body_P(p) = \{B_1, \dots, B_m\})$$

- Example

$$\frac{\begin{array}{c}\mathbf{F}\{b, {\sim}c\}\\ \mathbf{F}\{d, {\sim}e\}\end{array}}{\mathbf{F}a} \;(body_P(a) = \{\{b, {\sim}c\}, \{d, {\sim}e\}\})$$

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- Example

$$\frac{\begin{array}{c}\mathbf{F}\{b,{\sim}c\}\\ \mathbf{F}\{d,{\sim}e\}\end{array}}{\mathbf{F}a}\ (body_P(a)=\{\{b,{\sim}c\},\{d,{\sim}e\}\})$$

Backward True Atom (BTA)

- Prerequisites An atom is *true*, and the bodies of all rules with the atom as head except for one are *false*
- Consequence The residual body is *true*
- Tableau Rule BTA

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\quad (body_P(p)=\{B_1,\dots,B_m\})$$

- Examples

$$\frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{b,{\sim}c\}\end{array}}{\mathbf{T}\{d,{\sim}e\}}\;(*)\qquad\qquad \frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{d,{\sim}e\}\end{array}}{\mathbf{T}\{b,{\sim}c\}}\;(*)$$

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- Examples

$$\frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{b,\sim c\}\end{array}}{\mathbf{T}\{d,\sim e\}}\ (*) \qquad \frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{d,\sim e\}\end{array}}{\mathbf{T}\{b,\sim c\}}\ (*)$$

$$(*)\quad body_P(a)=\{\{b,\sim c\},\{d,\sim e\}\}$$

Tableau rules for atoms

Consider an atom p such that $body_P(p) = \{B_1, \ldots, B_m\}$

- Rules FTA and BFA amount to implication

  $$B_1 \vee \cdots \vee B_m \rightarrow p$$

- Rules FFA and BTA amount to implication

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- Together they yield

  $$p \equiv B_1 \vee \cdots \vee B_m$$

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Relationship with program completion

Let P be a normal logic program

- The eight tableau rules introduced so far essentially provide (straightforward) inferences from $CF(P)$

Preliminaries for unfounded sets

Let P be a normal logic program

- For $P' \subseteq P$, define the greatest unfounded set of P wrt P' as

$$\mathbf{U}_P(P') = atom(P) \setminus Cn((P')^{\emptyset})$$

- For a loop $L \in loop(P)$, define the external bodies of L as

$$EB_P(L) = \{body(r) \mid r \in P, head(r) \in L, body(r)^+ \cap L = \emptyset\}$$

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Well-Founded Negation (WFN)

- Prerequisites An atom is in the greatest unfounded set wrt rules whose bodies are *false*
- Consequence The atom is *false*
- Tableau Rule WFN

$$\frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (p \in \mathbf{U}_P(\{r \in P \mid body(r) \notin \{B_1,\dots,B_m\}\}))$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a}\ (*) \qquad \frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a}\ (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow \sim b\})$$

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- Examples

$$\frac{\begin{array}{c} a \leftarrow {\sim}b \\ \mathbf{F}\{{\sim}b\} \end{array}}{\mathbf{F}a} \; (*) \qquad\qquad \frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow {\sim}b \\ \mathbf{F}\{{\sim}b\} \end{array}}{\mathbf{F}a} \; (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow {\sim}b\})$$

Well-Founded Justification (WFJ)

- Prerequisites A *true* atom is in the greatest unfounded set wrt rules whose bodies are *false*, if a particular body is made *false*
- Consequence The respective body is *true*
- Tableau Rule WFJ

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\quad (p \in \mathbf{U}_P(\{r \in P \mid body(r) \notin \{B_1,\dots,B_m\}\}))$$

- Examples

$$\frac{\begin{array}{c}a \leftarrow \sim b\\ \mathbf{T}a\end{array}}{\mathbf{T}\{\sim b\}}\ (*) \qquad \frac{\begin{array}{c}a \leftarrow a\\ a \leftarrow \sim b\\ \mathbf{T}a\end{array}}{\mathbf{T}\{\sim b\}}\ (*)$$

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- Examples

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Well-founded tableau rules

- Tableau rules WFN and WFJ ensure non-circular support for *true* atoms
- Note
  1. WFN subsumes falsifying atoms via FFA,
  2. WFJ can be viewed as "backward propagation" for unfounded sets,
  3. WFJ subsumes backward propagation of *true* atoms via BTA

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Relationship with well-founded operator

Let P be a normal logic program, $\langle T, F\rangle$ a partial interpretation, and $P' = \{r \in P \mid body(r)^+ \cap F = \emptyset, body(r)^- \cap T = \emptyset\}$.

- The following conditions are equivalent
  1. $p \in \mathbf{U}_P\langle T, F\rangle$
  2. $p \in \mathbf{U}_P(P')$
- Hence, the well-founded operator $\mathbf{\Omega}$ and WFN coincide
- Note In contrast to $\mathbf{\Omega}$, WFN does not necessarily require a rule body to contain a *false* literal for the rule being inapplicable

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Forward Loop (FL)

- Prerequisites The external bodies of a loop are *false*
- Consequence The atoms in the loop are *false*
- Tableau Rule FL

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- Prerequisites An atom of a loop is *true*, and all external bodies except for one are *false*
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Tableau rules for loops

- Tableau rules FL and BL ensure non-circular support for *true* atoms
- For a loop L such that $EB_P(L) = \{B_1, \dots, B_m\}$, they amount to implications of form

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- Tableau rules FL and BL essentially provide (straightforward) inferences from loop formulas
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Well-known tableau calculi

- Fitting's operator $\mathbf{\Phi}$ applies forward propagation without sophisticated unfounded set checks

$$\mathcal{T}_{\mathbf{\Phi}} = \{FTB, FTA, FFB, FFA\}$$

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- SAT-based ASP solvers, *assat* and *cmodels*, incrementally add loop formulas to a program's completion
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Proof complexity

- Proof complexity is used for describing the relative efficiency of different proof systems
  - It compares proof systems based on minimal refutations
  - It is independent of heuristics
- A proof system $\mathcal{T}$ polynomially simulates a proof system $\mathcal{T}'$, if every refutation of $\mathcal{T}'$ can be polynomially mapped to a refutation of $\mathcal{T}$
  - Otherwise, $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$
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$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

- $\mathcal{T}_{smodels}$ restricts Cut to $atom(P)$ and $\mathcal{T}_{noMoRe}$ to $body(P)$
  Are both approaches similar or is one of them superior to the other?
- Let $\{P_a^n\}$, $\{P_b^n\}$, and $\{P_c^n\}$ be infinite families of programs where

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- $\mathcal{T}_{smodels}$ restricts Cut to $atom(P)$ and $\mathcal{T}_{noMoRe}$ to $body(P)$
  Are both approaches similar or is one of them superior to the other?

- Let $\{P_a^n\}$, $\{P_b^n\}$, and $\{P_c^n\}$ be infinite families of programs where

$$P_a^n = \left\{ \begin{array}{c} x \leftarrow \sim x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P_b^n = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, \sim x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P_c^n = \left\{ \begin{array}{c} a_1 \leftarrow \sim b_1 \\ b_1 \leftarrow \sim a_1 \\ \vdots \\ a_n \leftarrow \sim b_n \\ b_n \leftarrow \sim a_n \end{array} \right\}$$

- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $Cut[body(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P_a^n \cup P_c^n)]$
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Relative efficiency

- As witnessed by $\{P_a^n \cup P_c^n\}$ and $\{P_b^n \cup P_c^n\}$, respectively, $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ do not polynomially simulate one another
- Any refutation of $\mathcal{T}_{smodels}$ or $\mathcal{T}_{noMoRe}$ is a refutation of $\mathcal{T}_{nomore^{++}}$ (but not vice versa)
- Hence
  - both $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ are polynomially simulated by $\mathcal{T}_{nomore^{++}}$ and
  - $\mathcal{T}_{nomore^{++}}$ is polynomially simulated by neither $\mathcal{T}_{smodels}$ nor $\mathcal{T}_{noMoRe}$
- More generally, the proof system obtained with $Cut[atom(P) \cup body(P)]$ is exponentially stronger than the ones with either $Cut[atom(P)]$ or $Cut[body(P)]$
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$\mathcal{T}_{smodels}$: Example tableau

(r_1)	$a \leftarrow {\sim}b$		(r_2)	$b \leftarrow d, {\sim}a$		(r_3)	$c \leftarrow b, d$	
(r_4)	$c \leftarrow g$		(r_5)	$d \leftarrow c$		(r_6)	$d \leftarrow g$	
(r_7)	$e \leftarrow f, {\sim}c$		(r_8)	$f \leftarrow {\sim}g$		(r_9)	$g \leftarrow {\sim}a, {\sim}f$	

(1)	$\mathbf{T}a$	$[Cut]$
(2)	$\mathbf{T}\{{\sim}b\}$	$[BTA\colon r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB\colon 2]$
(4)	$\mathbf{F}\{d, {\sim}a\}$	$[BFA\colon r_2, 3]$
(5)	$\mathbf{F}\{{\sim}a, {\sim}f\}$	$[FFB\colon r_9, 1]$
(6)	$\mathbf{F}g$	$[FFA\colon r_9, 5]$
(7)	$\mathbf{T}\{{\sim}g\}$	$[FTB\colon r_8, 6]$
(8)	$\mathbf{T}f$	$[FTA\colon r_8, 7]$
(9)	$\mathbf{F}\{b, d\}$	$[FFB\colon r_3, 3]$
(10)	$\mathbf{F}\{g\}$	$[FFB\colon r_4, r_6, 6]$
(11)	$\mathbf{F}c$	$[FFA\colon r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB\colon r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA\colon r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, {\sim}c\}$	$[FTB\colon r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA\colon r_7, 14]$

(16)	$\mathbf{F}a$	$[Cut]$
(17)	$\mathbf{F}\{{\sim}b\}$	$[BFA\colon r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB\colon 17]$
(19)	$\mathbf{T}\{d, {\sim}a\}$	$[BTA\colon r_2, 18]$
(20)	$\mathbf{T}d$	$[BTB\colon 19]$
(21)	$\mathbf{T}\{b, d\}$	$[FTB\colon r_3, 18, 20]$
(22)	$\mathbf{T}c$	$[FTA\colon r_3, 21]$
(23)	$\mathbf{F}\{f, {\sim}c\}$	$[FFB\colon r_7, 22]$
(24)	$\mathbf{F}e$	$[FFA\colon r_7, 23]$
(25)	$\mathbf{T}\{c\}$	$[FTB\colon r_5, 22]$

(26)	$\mathbf{T}f$	$[Cut]$
(27)	$\mathbf{F}\{{\sim}a, {\sim}f\}$	$[FFB\colon r_9, 26]$
(28)	$\mathbf{F}c$	$[WFN\colon 27]$

(29)	$\mathbf{F}f$	$[Cut]$
(30)	$\mathbf{T}\{{\sim}a, {\sim}f\}$	$[FTB\colon r_9, 16, 29]$
(31)	$\mathbf{T}g$	$[FTA\colon r_9, 30]$
(32)	$\mathbf{T}\{g\}$	$[FTB\colon r_4, r_6, 31]$
(33)	$\mathbf{F}\{{\sim}g\}$	$[FFB\colon r_8, 31]$

$\mathcal{T}_{noMoRe}$: Example tableau

(r_1)	$a \leftarrow \sim b$	(r_2)	$b \leftarrow d, \sim a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, \sim c$	(r_8)	$f \leftarrow \sim g$	(r_9)	$g \leftarrow \sim a, \sim f$

(1)	$\mathbf{T}\{\sim b\}$	$[Cut]$
(2)	$\mathbf{T}a$	$[FTA\text{: } r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB\text{: } 1]$
(4)	$\mathbf{F}\{d, \sim a\}$	$[BFA\text{: } r_2, 3]$
(5)	$\mathbf{F}\{\sim a, \sim f\}$	$[FFB\text{: } r_9, 2]$
(6)	$\mathbf{F}g$	$[FFA\text{: } r_9, 5]$
(7)	$\mathbf{T}\{\sim g\}$	$[FTB\text{: } r_8, 6]$
(8)	$\mathbf{T}f$	$[FTA\text{: } r_8, 7]$
(9)	$\mathbf{F}\{b, d\}$	$[FFB\text{: } r_3, 3]$
(10)	$\mathbf{F}\{g\}$	$[FFB\text{: } r_4, r_6, 6]$
(11)	$\mathbf{F}c$	$[FFA\text{: } r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB\text{: } r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA\text{: } r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, \sim c\}$	$[FTB\text{: } r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA\text{: } r_7, 14]$

(16)	$\mathbf{F}\{\sim b\}$	$[Cut]$
(17)	$\mathbf{F}a$	$[FFA\text{: } r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB\text{: } 16]$
(19)	$\mathbf{T}\{d, \sim a\}$	$[BTA\text{: } r_2, 18]$
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(24)	$\mathbf{F}e$	$[FFA\text{: } r_7, 23]$
(25)	$\mathbf{T}\{c\}$	$[FTB\text{: } r_5, 22]$

(26)	$\mathbf{T}\{\sim g\}$	$[Cut]$
(27)	$\mathbf{F}g$	$[BTB\text{: } 26]$
(28)	$\mathbf{F}\{g\}$	$[FFB\text{: } r_4, r_6, 27]$
(29)	$\mathbf{F}c$	$[WFN\text{: } 28]$

(30)	$\mathbf{F}\{\sim g\}$	$[Cut]$
(31)	$\mathbf{T}g$	$[BFB\text{: } 30]$
(32)	$\mathbf{T}\{g\}$	$[FTB\text{: } r_4, r_6, 31]$
(33)	$\mathbf{F}f$	$[FFA\text{: } r_8, 30]$
(34)	$\mathbf{T}\{\sim a, \sim f\}$	$[FTB\text{: } r_9, 17, 33]$

$\mathcal{T}_{nomore^{++}}$: Example tableau

$$
\begin{array}{llllll}
(r_1) & a \leftarrow \sim b & (r_2) & b \leftarrow d, \sim a & (r_3) & c \leftarrow b, d \\
(r_4) & c \leftarrow g & (r_5) & d \leftarrow c & (r_6) & d \leftarrow g \\
(r_7) & e \leftarrow f, \sim c & (r_8) & f \leftarrow \sim g & (r_9) & g \leftarrow \sim a, \sim f
\end{array}
$$

(1)	$\mathbf{T}a$	[*Cut*]
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(16)	$\mathbf{F}a$	[*Cut*]
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(30)	$\mathbf{F}\{\sim g\}$	[*Cut*]
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Conflict-driven ASP Solving: Overview

36 Motivation

37 Boolean constraints

38 Nogoods from logic programs
- Nogoods from program completion
- Nogoods from loop formulas

39 Conflict-driven nogood learning
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Motivation

- Goal Approach to computing stable models of logic programs, based on concepts from
  - Constraint Processing (CP) and
  - Satisfiability Testing (SAT)
- Idea View inferences in ASP as unit propagation on nogoods
- Benefits
  - A uniform constraint-based framework for different kinds of inferences in ASP
  - Advanced techniques from the areas of CP and SAT
  - Highly competitive implementation

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Assignments

- An assignment A over $dom(A) = atom(P) \cup body(P)$ is a sequence

  $$(\sigma_1, \dots, \sigma_n)$$

  of signed literals σ_i of form $\mathbf{T}v$ or $\mathbf{F}v$ for $v \in dom(A)$ and $1 \leq i \leq n$
- $\mathbf{T}v$ expresses that v is *true* and $\mathbf{F}v$ that it is *false*
- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}v} = \mathbf{F}v$ and $\overline{\mathbf{F}v} = \mathbf{T}v$
- $A \circ \sigma$ stands for the result of appending σ to A
- Given $A = (\sigma_1, \dots, \sigma_{k-1}, \sigma_k, \dots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \dots, \sigma_{k-1})$
- We sometimes identify an assignment with the set of its literals
- Given this, we access *true* and *false* propositions in A via

  $$A^{\mathbf{T}} = \{v \in dom(A) \mid \mathbf{T}v \in A\} \text{ and } A^{\mathbf{F}} = \{v \in dom(A) \mid \mathbf{F}v \in A\}$$

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- Given this, we access *true* and *false* propositions in A via

  $$A^{\mathbf{T}} = \{v \in dom(A) \mid \mathbf{T}v \in A\} \text{ and } A^{\mathbf{F}} = \{v \in dom(A) \mid \mathbf{F}v \in A\}$$

Assignments

- An assignment A over $dom(A) = atom(P) \cup body(P)$ is a sequence

$$(\sigma_1, \ldots, \sigma_n)$$

  of signed literals σ_i of form $\mathbf{T}v$ or $\mathbf{F}v$ for $v \in dom(A)$ and $1 \leq i \leq n$
- $\mathbf{T}v$ expresses that v is *true* and $\mathbf{F}v$ that it is *false*
- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}v} = \mathbf{F}v$ and $\overline{\mathbf{F}v} = \mathbf{T}v$
- $A \circ \sigma$ stands for the result of appending σ to A
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Nogoods, solutions, and unit propagation

- A nogood is a set $\{\sigma_1, \dots, \sigma_n\}$ of signed literals, expressing a constraint violated by any assignment containing $\sigma_1, \dots, \sigma_n$
- An assignment A such that $A^{\mathbf{T}} \cup A^{\mathbf{F}} = dom(A)$ and $A^{\mathbf{T}} \cap A^{\mathbf{F}} = \emptyset$ is a solution for a set Δ of nogoods, if $\delta \not\subseteq A$ for all $\delta \in \Delta$
- For a nogood δ, a literal $\sigma \in \delta$, and an assignment A, we say that $\overline{\sigma}$ is unit-resulting for δ wrt A, if
  1. $\delta \setminus A = \{\sigma\}$ and
  2. $\overline{\sigma} \notin A$
- For a set Δ of nogoods and an assignment A, unit propagation is the iterated process of extending A with unit-resulting literals until no further literal is unit-resulting for any nogood in Δ

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- CDNL-ASP Algorithm
- Nogood Propagation
- Conflict Analysis

Outline

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Nogoods from logic programs
via program completion

The completion of a logic program P can be defined as follows:

$$\begin{aligned} & \{v_B \leftrightarrow a_1 \wedge \dots \wedge a_m \wedge \neg a_{m+1} \wedge \dots \wedge \neg a_n \mid \\ & \qquad B \in body(P), B = \{a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n\}\} \\ \cup \quad & \{a \leftrightarrow v_{B_1} \vee \dots \vee v_{B_k} \mid \\ & \qquad a \in atom(P), body(a) = \{B_1, \dots, B_k\}\} \ , \end{aligned}$$

where $body(a) = \{body(r) \mid r \in P, head(r) = a\}$

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \land \cdots \land a_m \land \neg a_{m+1} \land \cdots \land \neg a_n$$

can be decomposed into two implications:

1. $v_B \rightarrow a_1 \land \cdots \land a_m \land \neg a_{m+1} \land \cdots \land \neg a_n$
   is equivalent to the conjunction of

$$\neg v_B \lor a_1,\ \ldots,\ \neg v_B \lor a_m,\ \neg v_B \lor \neg a_{m+1},\ \ldots,\ \neg v_B \lor \neg a_n$$

   and induces the set of nogoods

$$\Delta(B) = \{\, \{\mathbf{T}B, \mathbf{F}a_1\}, \ldots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \ldots, \{\mathbf{T}B, \mathbf{T}a_n\} \,\}$$

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

2. $a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n \rightarrow v_B$

   gives rise to the nogood

$$\delta(B) = \{\mathbf{F}B, \mathbf{T}a_1, \ldots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \ldots, \mathbf{F}a_n\}$$

Nogoods from logic programs
via program completion

- Analogously, the (atom-oriented) equivalence

  $$a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k}$$

  yields the nogoods

  1. $\Delta(a) = \{\, \{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\} \,\}$ and
  2. $\delta(a) = \{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\}$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lcl} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
$\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}} \qquad \begin{array}{l} \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\} \\ \{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\} \end{array}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal
$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
$\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \dots, B_k\}$, we get

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- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

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$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal
- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

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Nogoods from logic programs
atom-oriented nogoods

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\, \{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\,\{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\}\,\}$$

- Example Given Body $\{x, \sim y\}$, we obtain

$$\boxed{\begin{array}{c} \dots \leftarrow x, \sim y \\ \vdots \\ \dots \leftarrow x, \sim y \end{array}}$$

$$\{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\,\{\mathbf{T}\{x, \sim y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \sim y\}, \mathbf{T}y\}\,\}$$

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- $\mathbf{T}\{x, \sim y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
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Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \ldots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \ldots, \mathbf{F}a_n\}$$
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- Example Given Body $\{x, {\sim}y\}$, we obtain

$$\boxed{\begin{array}{c} \ldots \leftarrow x, {\sim}y \\ \vdots \\ \ldots \leftarrow x, {\sim}y \end{array}}$$

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Characterization of stable models

for tight logic programs

Let P be a logic program and

$$\begin{array}{rcl}\Delta_P & = & \{\delta(a) \mid a \in atom(P)\} \cup \{\delta \in \Delta(a) \mid a \in atom(P)\} \\ & \cup & \{\delta(B) \mid B \in body(P)\} \cup \{\delta \in \Delta(B) \mid B \in body(P)\}\end{array}$$

Theorem

Let P be a tight logic program. Then,
$X \subseteq atom(P)$ is a stable model of P iff
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Characterization of stable models

for tight logic programs, ie. free of positive recursion

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Nogoods from logic programs
via loop formulas

Let P be a normal logic program and recall that:

- For $L \subseteq atom(P)$, the external supports of L for P are

$$ES_P(L) = \{r \in P \mid head(r) \in L, body(r)^+ \cap L = \emptyset\}$$

- The (disjunctive) loop formula of L for P is

$$\begin{aligned} LF_P(L) &= \left(\bigvee_{A \in L} A\right) \rightarrow \left(\bigvee_{r \in ES_P(L)} body(r)\right) \\ &\equiv \left(\bigwedge_{r \in ES_P(L)} \neg body(r)\right) \rightarrow \left(\bigwedge_{A \in L} \neg A\right) \end{aligned}$$

  - Note The loop formula of L enforces all atoms in L to be *false* whenever L is not externally supported

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Nogoods from logic programs
loop nogoods

- For a logic program P and some $\emptyset \subset U \subseteq atom(P)$, define the loop nogood of an atom $a \in U$ as
$$\lambda(a, U) \quad = \quad \{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\}$$
where $EB_P(U) = \{B_1, \dots, B_k\}$
- We get the following set of loop nogoods for P:
$$\Lambda_P \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(P)} \{\lambda(a, U) \mid a \in U\}$$
- The set Λ_P of loop nogoods denies cyclic support among *true* atoms

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Example

- Consider the program

$$\left\{ \begin{array}{ll} x \leftarrow \sim y & u \leftarrow x \\ & u \leftarrow v \\ y \leftarrow \sim x & v \leftarrow u, y \end{array} \right\}$$

- For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) \;=\; \{\mathbf{T}u, \mathbf{F}\{x\}\}$$

Similarly for v in $\{u, v\}$, we get:

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Characterization of stable models

Theorem

Let P be a logic program. Then,
$X \subseteq atom(P)$ *is a stable model of* P *iff*
$X = A^{\mathbf{T}} \cap atom(P)$ *for a (unique) solution* A *for* $\Delta_P \cup \Lambda_P$

Some remarks

- Nogoods in Λ_P augment Δ_P with conditions checking for unfounded sets, in particular, those being loops
- While $|\Delta_P|$ is linear in the size of P, Λ_P may contain exponentially many (non-redundant) loop nogoods

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Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
  - (Unit) propagation
  - (Chronological) backtracking
  - in ASP, eg *smodels*
- Modern CDCL-style approach
  (CDCL stands for 'Conflict-Driven Constraint Learning')
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion
  - in ASP, eg *clasp*

DPLL-style solving

loop

    propagate // deterministically assign literals

    if no conflict **then**

        if all variables assigned **then return** solution

        else *decide* // non-deterministically assign some literal

    else

        if top-level conflict **then return** unsatisfiable

        else

            backtrack // unassign literals made after last decision

            flip // assign complement of last decision literal

CDCL-style solving

loop

propagate // deterministically assign literals

if no conflict **then**

if all variables assigned **then return** solution
else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable
else

analyze // analyze conflict and add conflict constraint
backjump // unassign literals until conflict constraint is unit

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Outline of CDNL-ASP algorithm

- Keep track of deterministic consequences by unit propagation on:
  - Program completion [Δ_P]
  - Loop nogoods, determined and recorded on demand [Λ_P]
  - Dynamic nogoods, derived from conflicts and unfounded sets [∇]
- When a nogood in $\Delta_P \cup \nabla$ becomes violated:
  - Analyze the conflict by resolution (until reaching a Unique Implication Point, short: UIP)
  - Learn the derived conflict nogood δ
  - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
  - Assert the complement of the UIP and proceed (by unit propagation)
- Terminate when either:
  - Finding a stable model (a solution for $\Delta_P \cup \Lambda_P$)
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Algorithm 1: CDNL-ASP

Input : A normal program P
Output : A stable model of P or "no stable model"

$A := \emptyset$ // *assignment over atom(P) ∪ body(P)*
$\nabla := \emptyset$ // *set of recorded nogoods*
$dl := 0$ // *decision level*

loop
- $(A, \nabla) := \textsc{NogoodPropagation}(P, \nabla, A)$
- **if** $\varepsilon \subseteq A$ **for some** $\varepsilon \in \Delta_P \cup \nabla$ **then** // *conflict*
  - **if** $\max(\{dlevel(\sigma) \mid \sigma \in \varepsilon\} \cup \{0\}) = 0$ **then return** no stable model
  - $(\delta, dl) := \textsc{ConflictAnalysis}(\varepsilon, P, \nabla, A)$
  - $\nabla := \nabla \cup \{\delta\}$ // *(temporarily) record conflict nogood*
  - $A := A \setminus \{\sigma \in A \mid dl < dlevel(\sigma)\}$ // *backjumping*
- **else if** $A^{\mathbf{T}} \cup A^{\mathbf{F}} = atom(P) \cup body(P)$ **then** // *stable model*
  - **return** $A^{\mathbf{T}} \cap atom(P)$
- **else**
  - $\sigma_d := \textsc{Select}(P, \nabla, A)$ // *decision*
  - $dl := dl + 1$
  - $dlevel(\sigma_d) := dl$
  - $A := A \circ \sigma_d$

Observations

- Decision level dl, initially set to 0, is used to count the number of heuristically chosen literals in assignment A
- For a heuristically chosen literal $\sigma_d = \mathbf{T}a$ or $\sigma_d = \mathbf{F}a$, respectively, we require $a \in (atom(P) \cup body(P)) \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
- For any literal $\sigma \in A$, $dl(\sigma)$ denotes the decision level of σ, viz. the value dl had when σ was assigned
- A conflict is detected from violation of a nogood $\varepsilon \subseteq \Delta_P \cup \nabla$
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of stable models
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$
  - After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation
  - No explicit flipping of heuristically chosen literals !

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Example: CDNL-ASP

Consider

$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
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Example: CDNL-ASP

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37 Boolean constraints

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Outline of NogoodPropagation

- Derive deterministic consequences via:
  - Unit propagation on Δ_P and ∇;
  - Unfounded sets $U \subseteq atom(P)$
- Note that U is unfounded if $EB_P(U) \subseteq A^{\mathbf{F}}$
  - Note For any $a \in U$, we have $(\lambda(a, U) \setminus \{\mathbf{T}a\}) \subseteq A$
- An "interesting" unfounded set U satisfies:

$$\emptyset \subset U \subseteq (atom(P) \setminus A^{\mathbf{F}})$$

- Wrt a fixpoint of unit propagation, such an unfounded set contains some loop of P
  - Note Tight programs do not yield "interesting" unfounded sets !
- Given an unfounded set U and some $a \in U$, adding $\lambda(a, U)$ to ∇ triggers a conflict or further derivations by unit propagation
  - Note Add loop nogoods atom by atom to eventually falsify all $a \in U$

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Algorithm 2: NogoodPropagation

Input : A normal program P, a set ∇ of nogoods, and an assignment A.
Output : An extended assignment and set of nogoods.

$U := \emptyset$ // *unfounded set*

loop
  repeat
    if $\delta \subseteq A$ **for some** $\delta \in \Delta_P \cup \nabla$ **then return** (A, ∇) // *conflict*
    $\Sigma := \{\delta \in \Delta_P \cup \nabla \mid \delta \setminus A = \{\overline{\sigma}\}, \sigma \notin A\}$ // *unit-resulting nogoods*
    if $\Sigma \neq \emptyset$ **then let** $\overline{\sigma} \in \delta \setminus A$ **for some** $\delta \in \Sigma$ **in**
      $\mathit{dlevel}(\sigma) := \max(\{\mathit{dlevel}(\rho) \mid \rho \in \delta \setminus \{\overline{\sigma}\}\} \cup \{0\})$
      $A := A \circ \sigma$
  until $\Sigma = \emptyset$
  if $\mathit{loop}(P) = \emptyset$ **then return** (A, ∇)
  $U := U \setminus A^{\mathbf{F}}$
  if $U = \emptyset$ **then** $U :=$ UnfoundedSet(P, A)
  if $U = \emptyset$ **then return** (A, ∇) // *no unfounded set* $\emptyset \subset U \subseteq \mathit{atom}(P) \setminus A^{\mathbf{F}}$
  let $a \in U$ **in**
    $\nabla := \nabla \cup \{\{\mathbf{T}a\} \cup \{\mathbf{F}B \mid B \in EB_P(U)\}\}$ // *record loop nogood*

Requirements for UNFOUNDEDSET

- Implementations of UNFOUNDEDSET must guarantee the following for a result U
  1. $U \subseteq (atom(P) \setminus A^{\mathbf{F}})$
  2. $EB_P(U) \subseteq A^{\mathbf{F}}$
  3. $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(P) \setminus A^{\mathbf{F}})$
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  - Calculating the greatest unfounded set
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Example: NogoodPropagation

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$$P = \left\{ \begin{array}{llll} x \leftarrow {\sim}y & u \leftarrow x, y & v \leftarrow x & w \leftarrow {\sim}x, {\sim}y \\ y \leftarrow {\sim}x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

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Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood $\delta \in \Delta_P \cup \nabla$ becomes violated, viz. $\delta \subseteq A$, at a decision level $dl > 0$
  - Note that all but the first literal assigned at *dl* have been unit-resulting for nogoods $\varepsilon \in \Delta_P \cup \nabla$
  - If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$$

- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$
  - Iterated resolution progresses in inverse order of assignment
- Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level *dl*
  - This literal σ is called First Unique Implication Point (First-UIP)
  - All literals in $(\delta \setminus \{\sigma\})$ are assigned at decision levels smaller than *dl*

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Algorithm 3: ConflictAnalysis

Input : A non-empty violated nogood δ, a normal program P, a set ∇ of nogoods, and an assignment A.

Output : A derived nogood and a decision level.

loop
    let $\sigma \in \delta$ **such that** $\delta \setminus A[\sigma] = \{\sigma\}$ **in**
        $k := \max(\{dlevel(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\})$
        if $k = dlevel(\sigma)$ **then**
            let $\varepsilon \in \Delta_P \cup \nabla$ **such that** $\varepsilon \setminus A[\sigma] = \{\overline{\sigma}\}$ **in**
                $\delta := (\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$ *// resolution*
        else return (δ, k)

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$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ	
1	$\mathbf{T}u$			
2	$\mathbf{F}\{\sim x, \sim y\}$			
		$\mathbf{F}w$	$\{\mathbf{T}w, \mathbf{F}\{\sim x, \sim y\}\} = \delta(w)$	
3	$\mathbf{F}\{\sim y\}$			
		$\mathbf{F}x$	$\{\mathbf{T}x, \mathbf{F}\{\sim y\}\} = \delta(x)$	
		$\mathbf{F}\{x\}$	$\{\mathbf{T}\{x\}, \mathbf{F}x\} \in \Delta(\{x\})$	$\{\mathbf{T}u, \mathbf{F}x\}$
		$\mathbf{F}\{x, y\}$	$\{\mathbf{T}\{x, y\}, \mathbf{F}x\} \in \Delta(\{x, y\})$	$\{\mathbf{T}u, \mathbf{F}x, \mathbf{F}\{x\}\}$
		$\mathbf{T}\{\sim x\}$	$\{\mathbf{F}\{\sim x\}, \mathbf{F}x\} = \delta(\{\sim x\})$	
		$\mathbf{T}y$	$\{\mathbf{F}\{\sim y\}, \mathbf{F}y\} = \delta(\{\sim y\})$	
		$\mathbf{T}\{v\}$	$\{\mathbf{T}u, \mathbf{F}\{x, y\}, \mathbf{F}\{v\}\} = \delta(u)$	
		$\mathbf{T}\{u, y\}$	$\{\mathbf{F}\{u, y\}, \mathbf{T}u, \mathbf{T}y\} = \delta(\{u, y\})$	
		$\mathbf{T}v$	$\{\mathbf{F}v, \mathbf{T}\{u, y\}\} \in \Delta(v)$	
			$\{\mathbf{T}u, \mathbf{F}\{x\}, \mathbf{F}\{x, y\}\} = \lambda(u, \{u, v\})$	✘

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Remarks

- There always is a First-UIP at which conflict analysis terminates
  - In the worst, resolution stops at the heuristically chosen literal assigned at decision level dl
- The nogood δ containing First-UIP σ is violated by A, viz. $\delta \subseteq A$
- We have $k = max(\{dl(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\}) < dl$
  - After recording δ in ∇ and backjumping to decision level k, $\overline{\sigma}$ is unit-resulting for δ !
  - Such a nogood δ is called asserting
- Asserting nogoods direct conflict-driven search into a different region of the search space than traversed before, without explicitly flipping any heuristically chosen literal !

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Systems: Overview

40 Potassco

41 gringo

42 clasp

43 Siblings
- claspfolio
- claspD
- hclasp
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potassco.sourceforge.net

Potassco, the Potsdam Answer Set Solving Collection, bundles tools for ASP developed at the University of Potsdam, for instance:

- Grounder gringo, lingo, pyngo
- Solver clasp, {a,h,pb,un}clasp, claspD, claspfolio, claspar, aspeed
- Grounder+Solver Clingo, iClingo, {ros}oClingo, Clingcon
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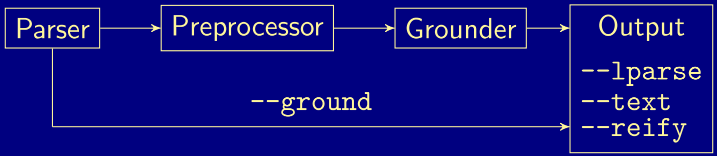
gringo

- Accepts safe programs with aggregates
- Tolerates unrestricted use of function symbols
  (as long as it yields a finite ground instantiation :)
- Expressive power of a Turing machine

- Basic architecture of *gringo*:



An example

$$
\begin{array}{l}
d(a) \\
d(c) \\
d(d) \\
p(a,b) \\
p(b,c) \\
p(c,d) \\
p(X,Z) \leftarrow p(X,Y), p(Y,Z) \\
q(a) \\
q(b) \\
q(X) \leftarrow \sim r(X), d(X) \\
r(X) \leftarrow \sim q(X), d(X) \\
s(X) \leftarrow \sim r(X), p(X,Y), q(Y)
\end{array}
$$

An example

Safe ?

$d(a)$
$d(c)$
$d(d)$
$p(a, b)$
$p(b, c)$
$p(c, d)$
$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$
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An example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a, b)$	✔
$p(b, c)$	✔
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$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$	
$q(a)$	✔
$q(b)$	✔
$q(X) \leftarrow \sim r(X), d(X)$	
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An example

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$r(X) \leftarrow \sim q(X), d(X)$	✔
$s(X) \leftarrow \sim r(X), p(X, Y), q(Y)$	✔

Match

- A substitution is a mapping from variables to terms
- Given sets B and D of atoms, a substitution θ is a match of B in D, if $B\theta \subseteq D$
- Given a set B of atoms and a set D of ground atoms, define

$$\Theta(B, D) = \{\, \theta \mid \theta \text{ is a } \subseteq\text{-minimal match of } B \text{ in } D \,\}$$

- Example $\{X \mapsto 1\}$ and $\{X \mapsto 2\}$ are $\subseteq$-minimal matches of $\{p(X)\}$ in $\{p(1), p(2), p(3)\}$, while match $\{X \mapsto 1, Y \mapsto 2\}$ is not

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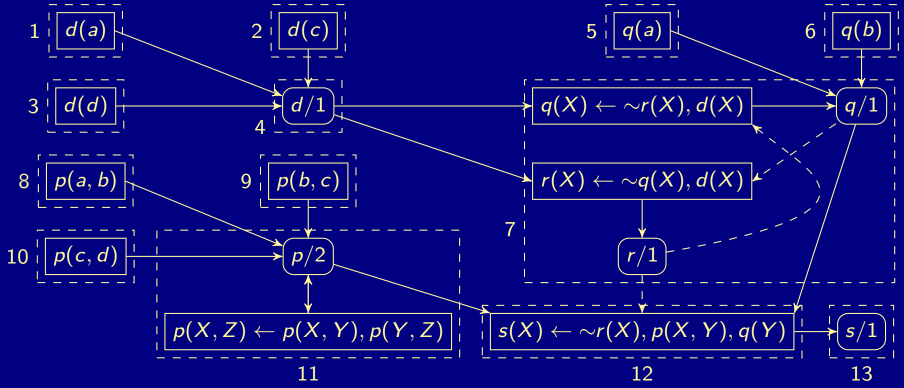
Naive instantiation

Algorithm 4: NaiveInstantiation

Input : A safe (first-order) logic program P
Output : A ground logic program P'

```
D := ∅
P' := ∅
repeat
    D' := D
    foreach r ∈ P do
        B := body(r)⁺
        foreach θ ∈ Θ(B, D) do
            D := D ∪ {head(r)θ}
            P' := P' ∪ {rθ}
until D = D'
```

Predicate-rule dependency graph



Instantiation

SCC	$\Theta(B, D)$	D	P'
1	$\{\emptyset\}$	$d(a)$	$d(a) \leftarrow$
2	$\{\emptyset\}$	$d(c)$	$d(c) \leftarrow$
3	$\{\emptyset\}$	$d(d)$	$d(d) \leftarrow$
5	$\{\emptyset\}$	$q(a)$	$q(a) \leftarrow$
6	$\{\emptyset\}$	$q(b)$	$q(b) \leftarrow$
7	$\{\{X \mapsto a\},$		$q(a) \leftarrow \sim r(a), d(a)$
	$\{X \mapsto c\},$	$q(c)$	$q(c) \leftarrow \sim r(c), d(c)$
	$\{X \mapsto d\},$	$q(d)$	$q(d) \leftarrow \sim r(d), d(d)$
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Instantiation

SCC	$\Theta(B, D)$	D	P'
8	$\{\emptyset\}$	$p(a, b)$	$p(a, b) \leftarrow$
9	$\{\emptyset\}$	$p(b, c)$	$p(b, c) \leftarrow$
10	$\{\emptyset\}$	$p(c, d)$	$p(c, d) \leftarrow$
11	$\{\{X \mapsto a, Y \mapsto b, Z \mapsto c\},$	$p(a, c)$	$p(a, c) \leftarrow p(a, b), p(b, c)$
	$\{X \mapsto b, Y \mapsto c, Z \mapsto d\}\}$	$p(b, d)$	$p(b, d) \leftarrow p(b, c), p(c, d)$
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	$\{X \mapsto a, Y \mapsto b, Z \mapsto d\}\}$		$p(a, d) \leftarrow p(a, b), p(b, d)$
12	$\{\{X \mapsto a, Y \mapsto b\},$	$s(a)$	$s(a) \leftarrow \sim r(a), p(a, b), q(b)$
	$\{X \mapsto a, Y \mapsto c\},$		$s(a) \leftarrow \sim r(a), p(a, c), q(c)$
	$\{X \mapsto a, Y \mapsto d\},$		$s(a) \leftarrow \sim r(a), p(a, d), q(d)$
	$\{X \mapsto b, Y \mapsto c\},$	$s(b)$	$s(b) \leftarrow \sim r(b), p(b, c), q(c)$
	$\{X \mapsto b, Y \mapsto d\},$		$s(b) \leftarrow \sim r(b), p(b, d), q(d)$
	$\{X \mapsto c, Y \mapsto d\}\}$	$s(c)$	$s(c) \leftarrow \sim r(c), p(c, d), q(d)$

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clasp

- *clasp* is a native ASP solver combining conflict-driven search with sophisticated reasoning techniques:
  - Advanced preprocessing including, like equivalence reasoning
  - lookback-based decision heuristics
  - restart policies
  - nogood deletion
  - progress saving
  - dedicated data structures for binary and ternary nogoods
  - lazy data structures (watched literals) for long nogoods
  - dedicated data structures for cardinality and weight constraints
  - lazy unfounded set checking based on “source pointers”
  - tight integration of unit propagation and unfounded set checking
  - various reasoning modes
  - parallel search
  - ...

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Reasoning modes of *clasp*

- Beyond deciding (stable) model existence, *clasp* allows for:
  - Optimization
  - Enumeration (without solution recording)
  - Projective enumeration (without solution recording)
  - Intersection and Union (linear solution computation)
  - and combinations thereof

- *clasp* allows for
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Parallel search in *clasp*

- *clasp*
  - pursues a coarse-grained, task-parallel approach to parallel search via shared memory multi-threading
    - up to 64 configurable (non-hierarchic) threads
  - allows for parallel solving via search space splitting and/or competing strategies
    - both supported by solver portfolios
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Sequential CDCL-style solving

loop

propagate // deterministically assign literals

if no conflict **then**

if all variables assigned **then return** solution

else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

else

analyze // analyze conflict and add conflict constraint

backjump // unassign literals until conflict constraint is unit

Parallel CDCL-style solving in *clasp*

while work available
  while no (result) message to send
    communicate // exchange information with other solver
    propagate // deterministically assign literals
    if no conflict **then**
      if all variables assigned **then send** solution
      else *decide* // non-deterministically assign some literal
    else
      if root-level conflict **then send** unsatisfiable
      else if external conflict **then send** unsatisfiable
      else
        analyze // analyze conflict and add conflict constraint
        backjump // unassign literals until conflict constraint is unit
  communicate // exchange results (and receive work)

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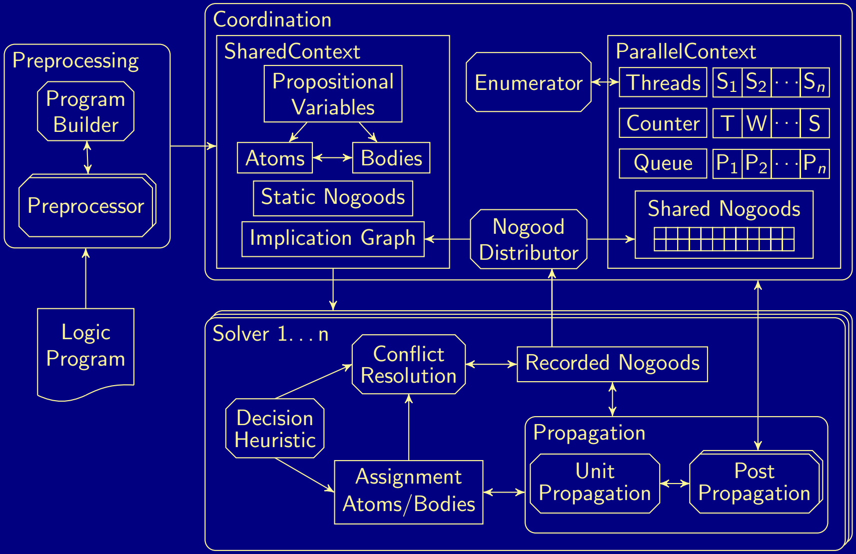
Multi-threaded architecture of *clasp*



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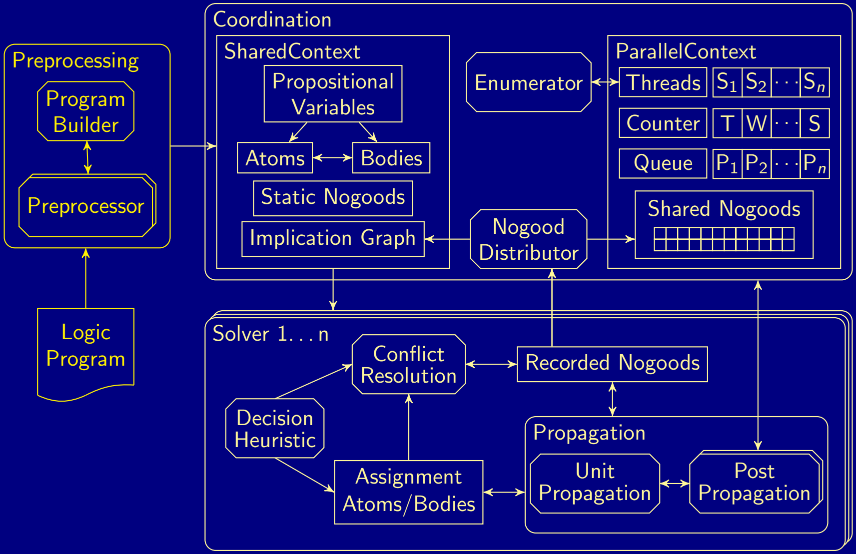

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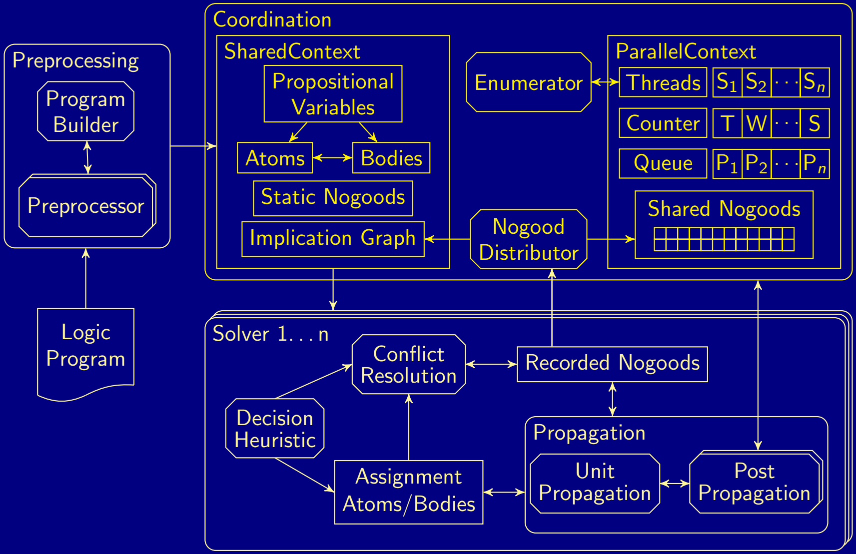

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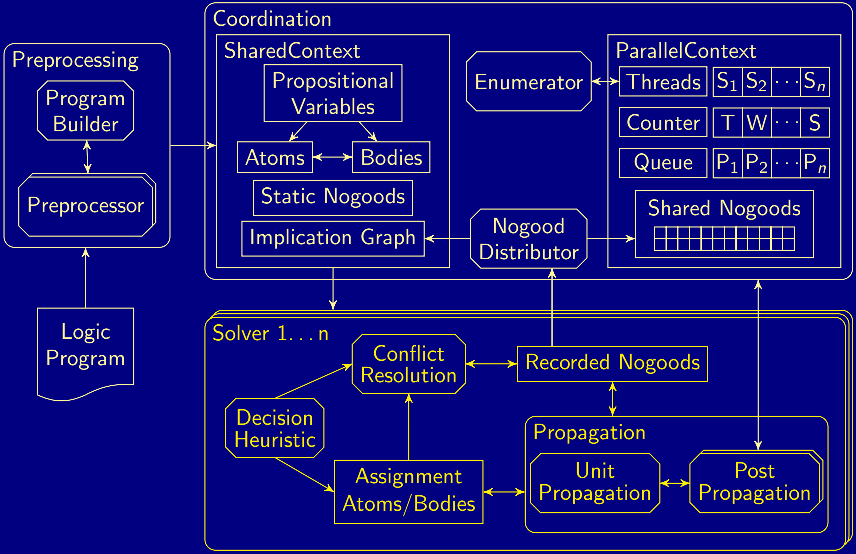


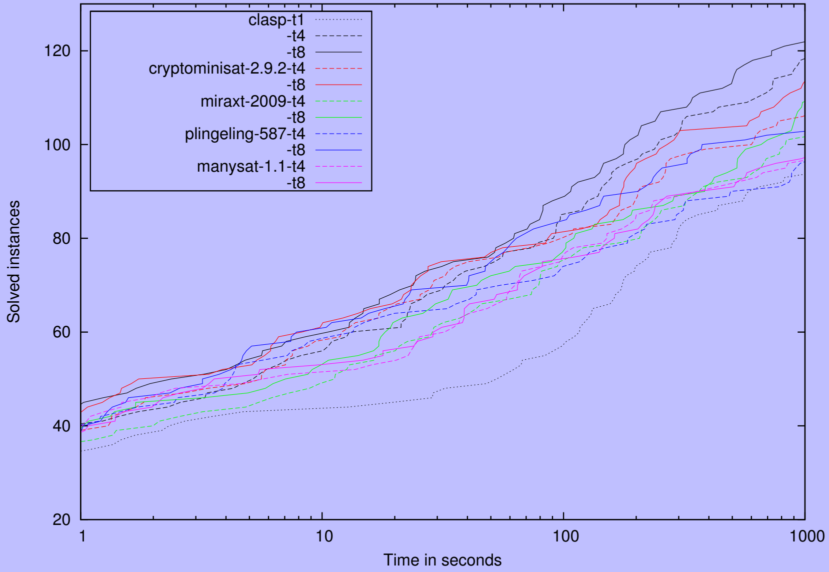
clasp in context

- Compare *clasp* (2.0.5) to the multi-threaded SAT solvers
  - *cryptominisat* (2.9.2)
  - *manysat* (1.1)
  - *miraxt* (2009)
  - *plingeling* (587f)

  all run with four and eight threads in their default settings

- 160/300 benchmarks from crafted category at SAT'11
  - all solvable by *ppfolio* in 1000 seconds
  - crafted SAT benchmarks are closest to ASP benchmarks

clasp in context



Using *clasp*

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--parallel-mode,-t <arg>: Run parallel search with given number of threads
    <arg>: <n {1..64}>[,<mode {compete|split}>]
      <n>   : Number of threads to use in search
      <mode>: Run competition or splitting based search [compete]

--configuration=<arg>   : Configure default configuration [frumpy]
    <arg>: {frumpy|jumpy|handy|crafty|trendy|chatty}
      frumpy: Use conservative defaults
      jumpy : Use aggressive defaults
      handy : Use defaults geared towards large problems
      crafty: Use defaults geared towards crafted problems
      trendy: Use defaults geared towards industrial problems
      chatty: Use 4 competing threads initialized via the default portfolio

--print-portfolio,-g    : Print default portfolio and exit
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Overview

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41 gringo

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Outline

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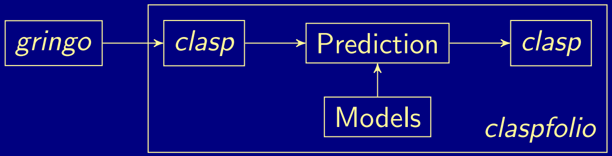
claspfolio

- Automatic selection of *clasp* configuration among 25 configuration via (learned) classifiers
- Basic architecture of *claspfolio*:



Solving with *clasp* (as usual)

```
$ clasp queens500 --quiet

clasp version 2.0.2
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 11.445s (Solving: 10.58s 1st Model: 10.55s Unsat: 0.00s)
CPU Time     : 11.410s
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Solving with *claspfolio*

```
$ claspfolio queens500 --quiet

PRESOLVING
Reading from queens500
Solving...
claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
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Models      : 1+
Time        : 4.785s (Solving: 3.96s 1st Model: 3.92s Unsat: 0.00s)
CPU Time    : 4.780s
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Feature-extraction with *claspfolio*

```
$ claspfolio --features queens500

PRESOLVING
Reading from queens500
Solving...
UNKNOWN
Features    : 84998,3994,0,250000,1.020,62.594,63.844,21.281,84998, \
 3994,100,250000,1.020,62.594,63.844,21.281,84998,3994,250,250000,  \
 1.020,62.594,63.844,21.281,84998,3994,475,250000,1.020,62.594,     \
 63.844,21.281,757989,757989,0,510983,506992,3990,1,0,127.066,9983, \
 1023958,502993,1994,518971,1,0,0,254994,0,3990,0.100,0.000,99.900, \
 0,270303,812,4,0,812,2223,2223,262,262,2.738,2.738,0.000,812,812,  \
 2270.982,0,0.000

$ claspfolio --list-features

maxLearnt,Constraints,LearntConstraints,FreeVars,Vars/FreeVars, ...
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Prediction with *claspfolio*

```
$ claspfolio queens500 --decisionvalues

PRESOLVING
Reading from queens500
Solving...

Portfolio Decision Values:
[1] : 3.437538   [10] : 3.639444   [19] : 3.726391
[2] : 3.501728   [11] : 3.483334   [20] : 3.020325
[3] : 3.784733   [12] : 3.271890   [21] : 3.220219
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UNKNOWN
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Solving with *claspfolio* (slightly verbosely)

```
$ claspfolio queens500 --quiet --autoverbose=1

PRESOLVING
Reading from queens500
Solving...

Chosen configuration: [20]
clasp --configurations=./models/portfolio.txt           \
      --modelpath=./models/                             \
      queens500 --quiet --autoverbose=1                 \
      --heu=VSIDS --sat-pre=20,25,120 --trans-ext=integ

claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 4.783s (Solving: 3.96s 1st Model: 3.93s Unsat: 0.00s)
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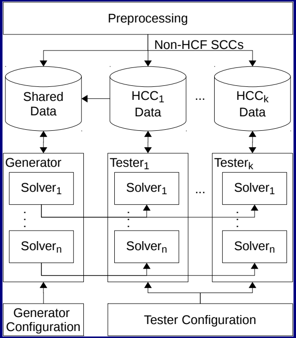
claspD

- *claspD* is a multi-threaded solver for disjunctive logic programs
- aiming at an equitable interplay between "generating" and "testing" solver units
- allowing for a bidirectional dynamic information exchange between solver units for orthogonal tasks



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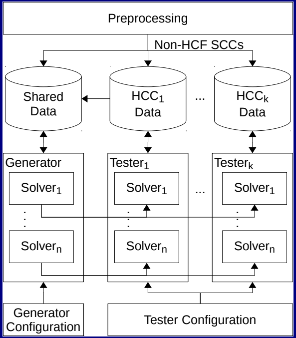

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hclasp

- *hclasp* allows for incorporating domain-specific heuristics into ASP solving
- Heuristic modifiers

  `init` for initializing the heuristic value of a with v,
  `factor` for amplifying the heuristic value of a by factor v,
  `level` for ranking all atoms; the rank of a is v,
  `sign` for attributing the sign of v as truth value to a.

- Example

```
_heuristics(occ(A,T),factor,T) :- action(A), time(T).
```

- The heuristic information is processed as an equitable part of the logic program and subsequently exploited by the solver when it comes to non-deterministically assigning a truth value to an atom.

hclasp

- *hclasp* allows for incorporating domain-specific heuristics into ASP solving
- Heuristic modifiers

`init`	for initializing the heuristic value of a with v,
`factor`	for amplifying the heuristic value of a by factor v,
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- The heuristic information is processed as an equitable part of the logic program and subsequently exploited by the solver when it comes to non-deterministically assigning a truth value to an atom.

hclasp

- *hclasp* allows for incorporating domain-specific heuristics into ASP solving
- Heuristic modifiers

`init`	for initializing the heuristic value of a with v,
`factor`	for amplifying the heuristic value of a by factor v,
`level`	for ranking all atoms; the rank of a is v,
`sign`	for attributing the sign of v as truth value to a.

- Example

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Outline

40 Potassco

41 gringo

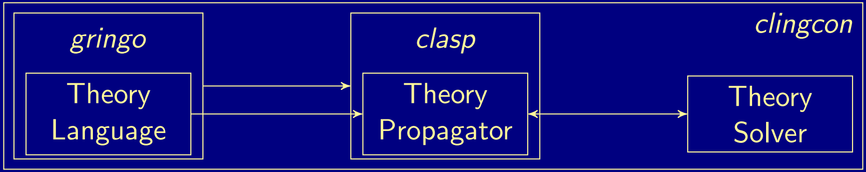
42 clasp

43 Siblings
- claspfolio
- claspD
- hclasp
- clingcon
- iclingo
- oclingo

clingcon

- Hybrid grounding and solving
- Solving in hybrid domains, like Bio-Informatics

- Basic architecture of *clingcon*:



Pouring Water into Buckets on a Scale

```
time(0..t).             $domain(0..500).
bucket(a).              volume(a,0) $== 0.
bucket(b).              volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

         1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30         :- pour(B,T), T < t.
amount(B,T) $==  0         :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).              $domain(0..500).
bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).             $domain(0..500).
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bucket(b).              volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
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bucket(a).                  volume(a,0) $== 0.
bucket(b).                  volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).              $domain(0..500).
bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, not (1 $<= amount(B,T)).
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
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bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
amount(B,T) $<= 30            :- pour(B,T), T < t.
amount(B,T) $==  0            :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
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bucket(a).              volume(a,0) $== 0.
bucket(b).              volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

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1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).                $domain(0..500).
bucket(a).                 volume(a,0) $== 0.
bucket(b).                 volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

 :- bucket(B), time(T), T < t, volume(B,T+1) $!= volume(B,T)$+amount(B,T).

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                        $domain(0..500).
bucket(a).                                                    :- volume(a,0) $!= 0.
bucket(b).                                                    :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                            ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                         ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                         ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                        ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                        ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.                    ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.                    ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).        ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).        ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).                 ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).                 ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).                 ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).                 ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                                ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                                ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                               $domain(0..500).
bucket(a).                                           :- volume(a,0) $!= 0.
bucket(b).                                           :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                   ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.               ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.               ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.           ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.           ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ... :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ... :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).        ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).        ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).        ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).        ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                       ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                       ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                 $domain(0..500).
bucket(a).                                             :- volume(a,0) $!= 0.
bucket(b).                                             :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                     ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                  ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                  ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                 ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                 ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.             ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.             ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).          ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).          ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).          ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).          ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                         ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                         ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                   $domain(0..500).
bucket(a).                                               :- volume(a,0) $!= 0.
bucket(b).                                               :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                      ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                   ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                   ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                  ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                  ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.              ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.              ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).           ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).           ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).           ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).           ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                          ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                          ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                       $domain(0..500).
bucket(a).                                                   :- volume(a,0) $!= 0.
bucket(b).                                                   :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                          ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                       ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                       ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                      ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                      ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.                  ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.                  ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).      ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).      ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).               ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).               ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).               ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).               ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                              ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                              ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0          1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0          1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0          1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0          1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100        volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100        volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100        volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100        volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100        volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1
Time         : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1
Time         : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0          1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0          1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0          1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0          1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100        volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100        volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100        volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100        volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100        volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Boolean variables

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Non-Boolean variables

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11         amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30         amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30         amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30         amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=11         volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=41         volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=71         volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=101        volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11         amount(b,0)=0          1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30         amount(b,1)=0          1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30         amount(b,2)=0          1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30         amount(b,3)=0          1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100        volume(a,0) $< volume(b,0)
volume(a,1)=11         volume(b,1)=100        volume(a,1) $< volume(b,1)
volume(a,2)=41         volume(b,2)=100        volume(a,2) $< volume(b,2)
volume(a,3)=71         volume(b,3)=100        volume(a,3) $< volume(b,3)
volume(a,4)=101        volume(b,4)=100        volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Outline

40 Potassco

41 gringo

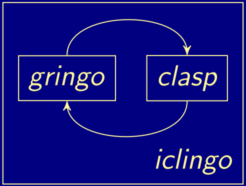
42 clasp

43 Siblings
- claspfolio
- claspD
- hclasp
- clingcon
- iclingo
- oclingo

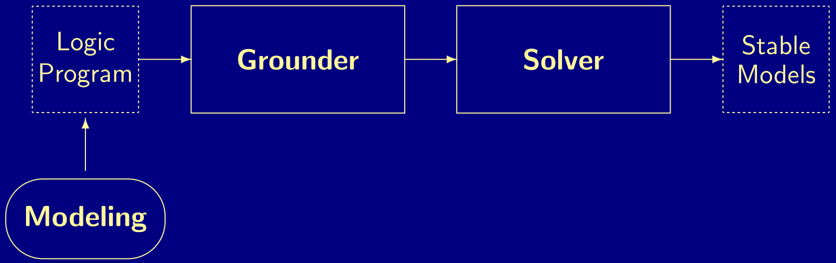
iclingo

- Incremental grounding and solving
- Offline solving in dynamic domains, like Automated Planning

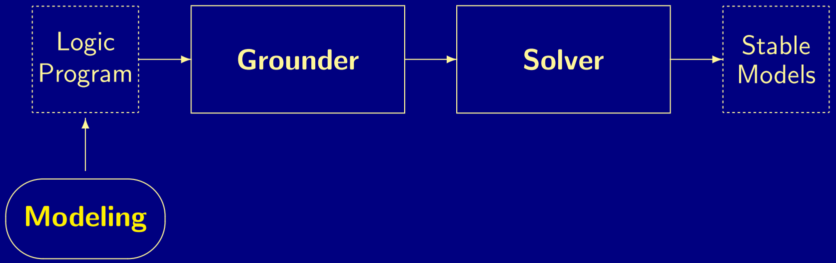
- Basic architecture of *iclingo*:



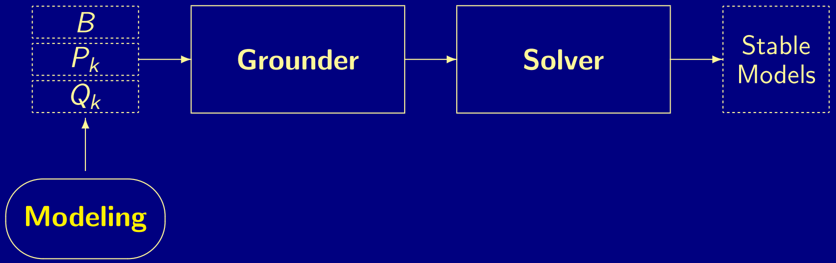
Incremental ASP Solving Process



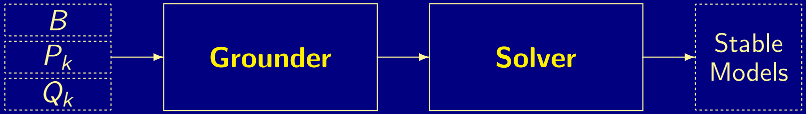
Incremental ASP Solving Process



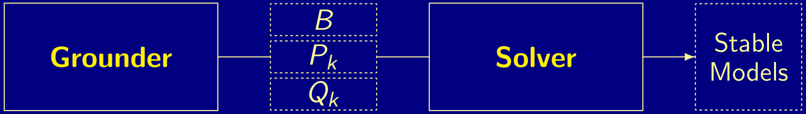
Incremental ASP Solving Process



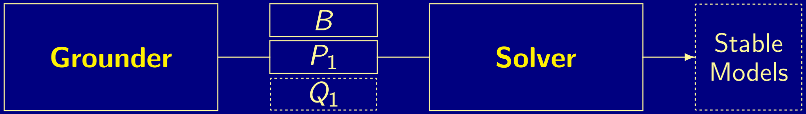
Incremental ASP Solving Process



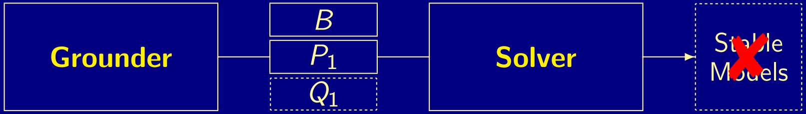
Incremental ASP Solving Process



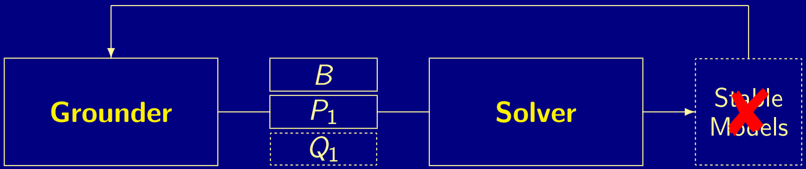
Incremental ASP Solving Process



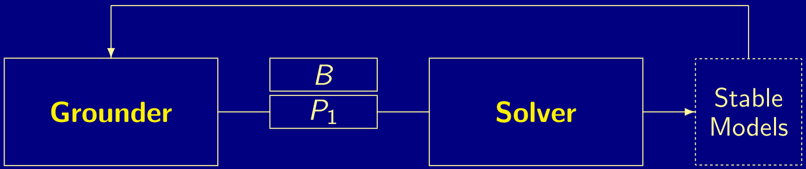
Incremental ASP Solving Process



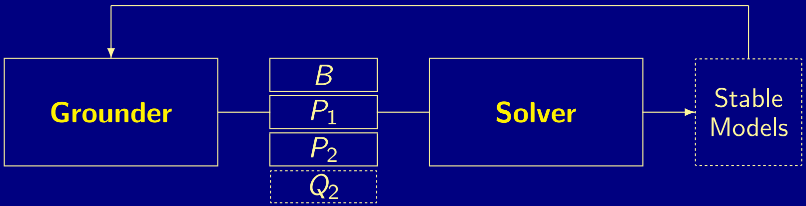
Incremental ASP Solving Process



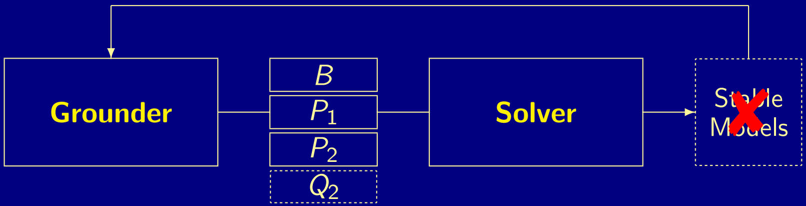
Incremental ASP Solving Process



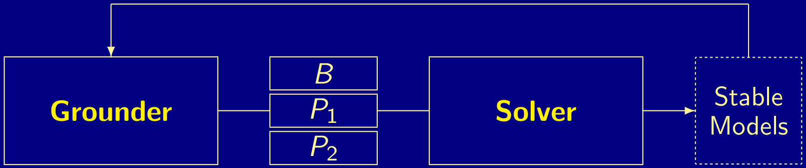
Incremental ASP Solving Process



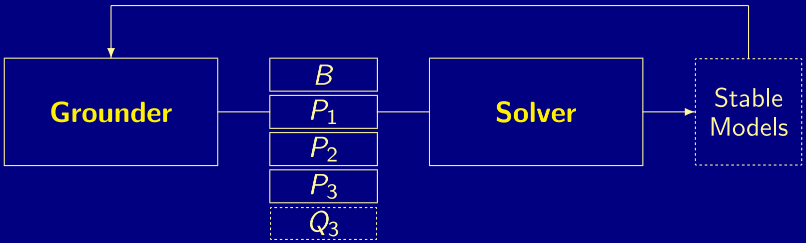
Incremental ASP Solving Process



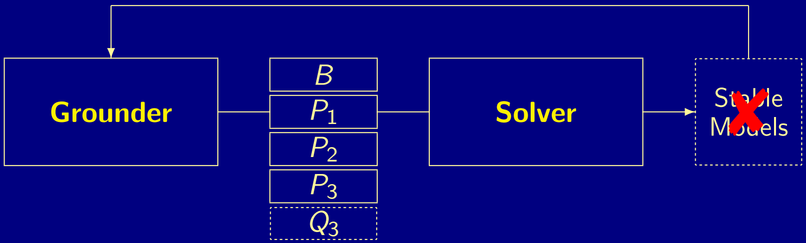
Incremental ASP Solving Process



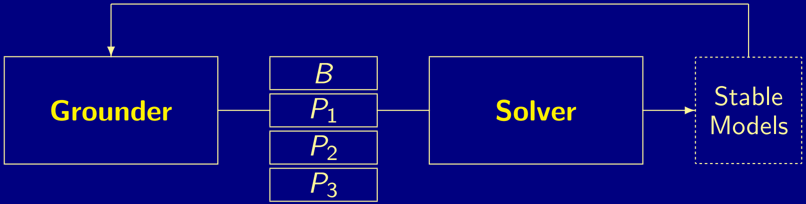
Incremental ASP Solving Process



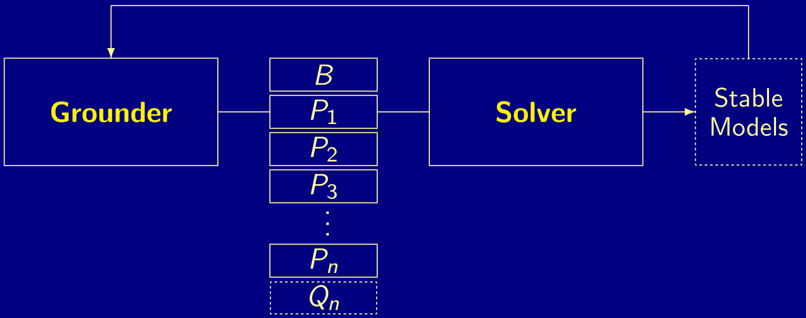
Incremental ASP Solving Process



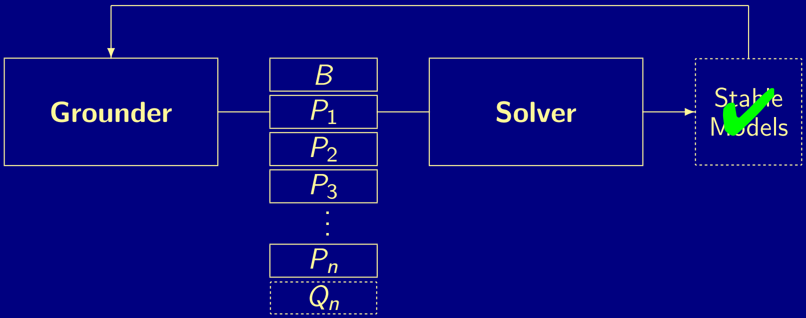
Incremental ASP Solving Process



Incremental ASP Solving Process



Incremental ASP Solving Process



Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

ocdel(F,t) :- occ(A,t), del(A,F).
holds(F,t) :- occ(A,t), add(A,F).
holds(F,t) :- holds(F,t-1), not ocdel(F,t).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).    action(a).    action(b).     init(p).
fluent(q).       pre(a,p).     pre(b,q).
fluent(r).       add(a,q).     add(b,r).   query(r).
                 del(a,p).     del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

ocdel(F,t) :- occ(A,t), del(A,F).
holds(F,t) :- occ(A,t), add(A,F).
holds(F,t) :- holds(F,t-1), not ocdel(F,t).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).     query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

ocdel(F,t) :- occ(A,t), del(A,F).
holds(F,t) :- occ(A,t), add(A,F).
holds(F,t) :- holds(F,t-1), not ocdel(F,t).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).     query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

ocdel(F,t) :- occ(A,t), del(A,F).
holds(F,t) :- occ(A,t), add(A,F).
holds(F,t) :- holds(F,t-1), not ocdel(F,t).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
=============== step 2 ===============
Answer: 1
occ(a,1) occ(b,2)

Models   : 1
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
=============== step 2 ===============
Answer: 1
occ(a,1) occ(b,2)

Models   : 1
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Outline

40 Potassco

41 gringo

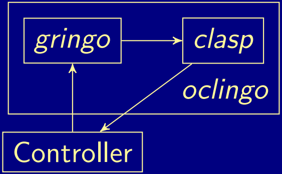
42 clasp

43 Siblings
- claspfolio
- claspD
- hclasp
- clingcon
- iclingo
- oclingo

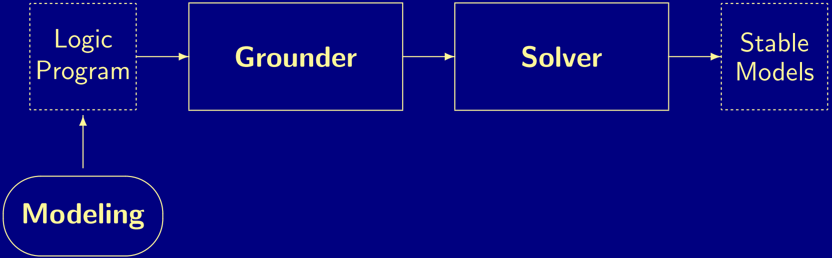
oclingo

- Reactive grounding and solving
- Online solving in dynamic domains, like Robotics

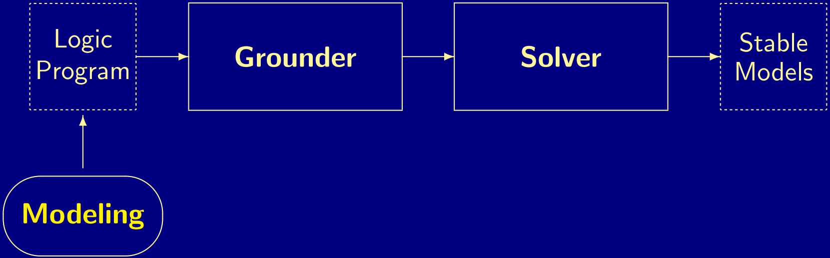
- Basic architecture of *oclingo*:



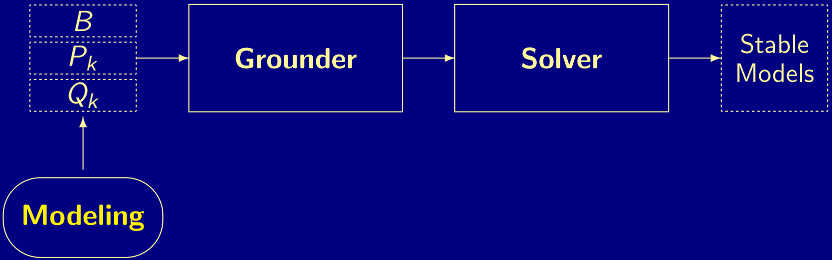
Reactive ASP Solving Process



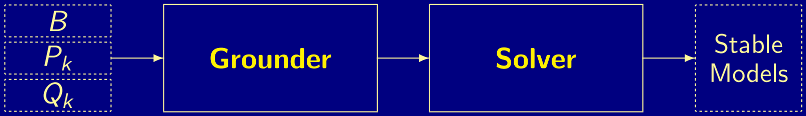
Reactive ASP Solving Process



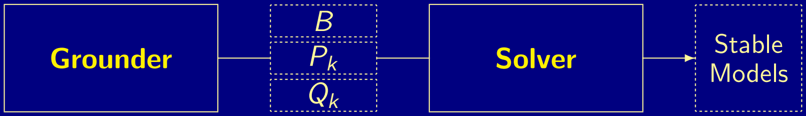
Reactive ASP Solving Process



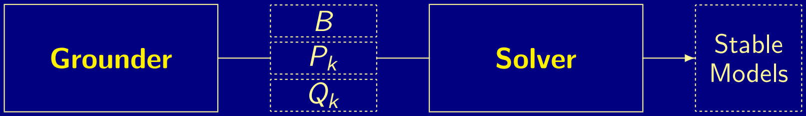
Reactive ASP Solving Process



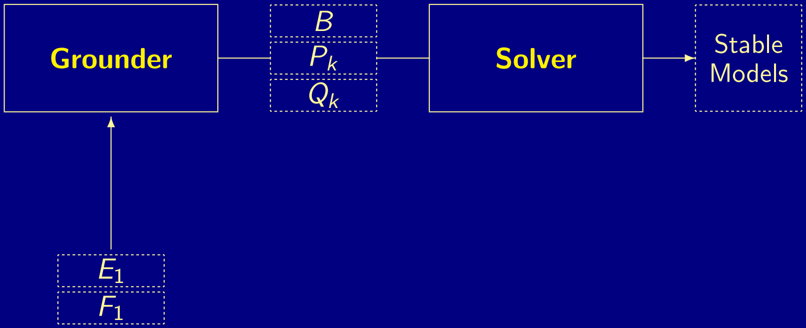
Reactive ASP Solving Process



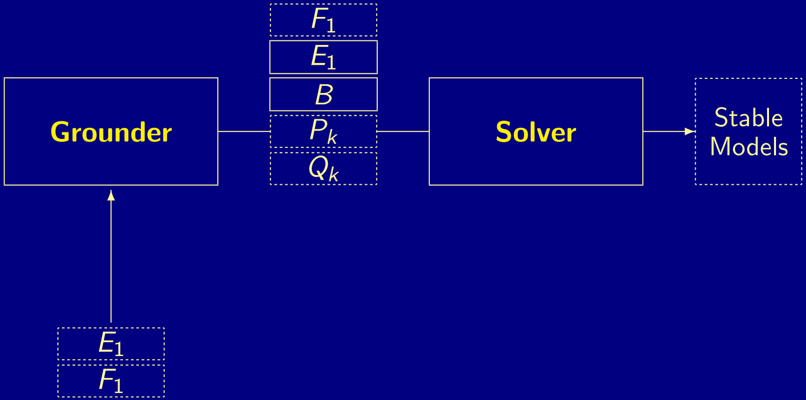
Reactive ASP Solving Process



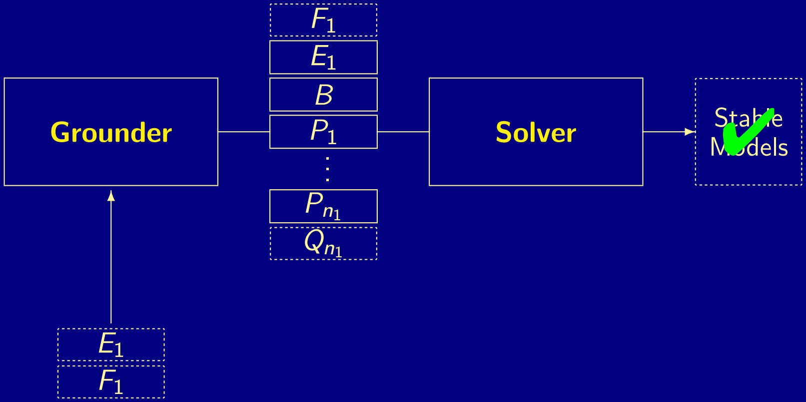
Reactive ASP Solving Process



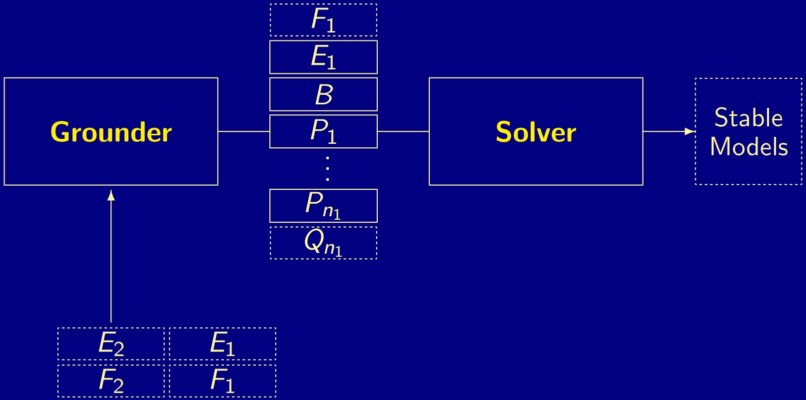
Reactive ASP Solving Process



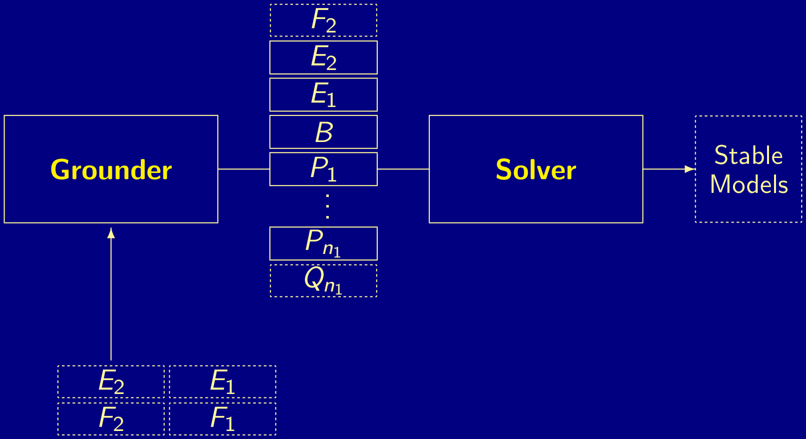
Reactive ASP Solving Process



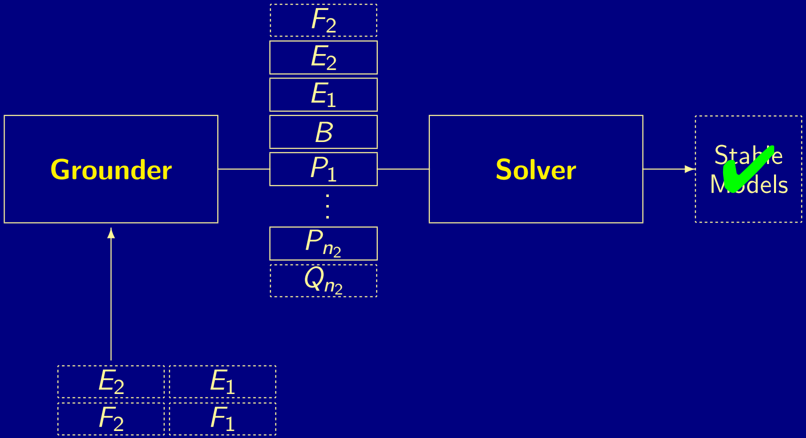
Reactive ASP Solving Process



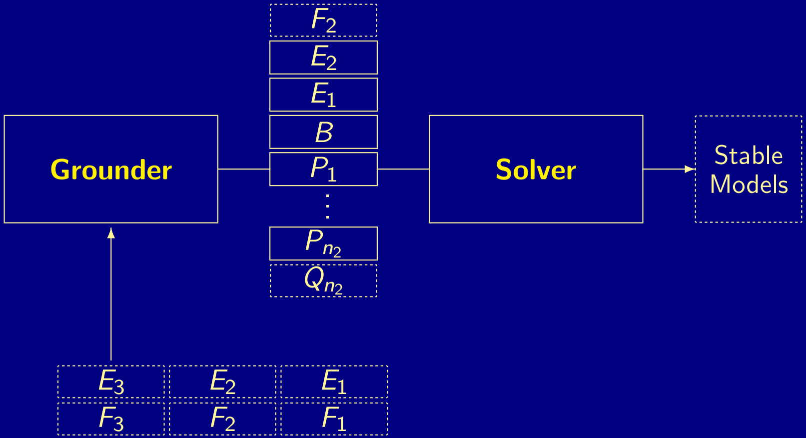
Reactive ASP Solving Process



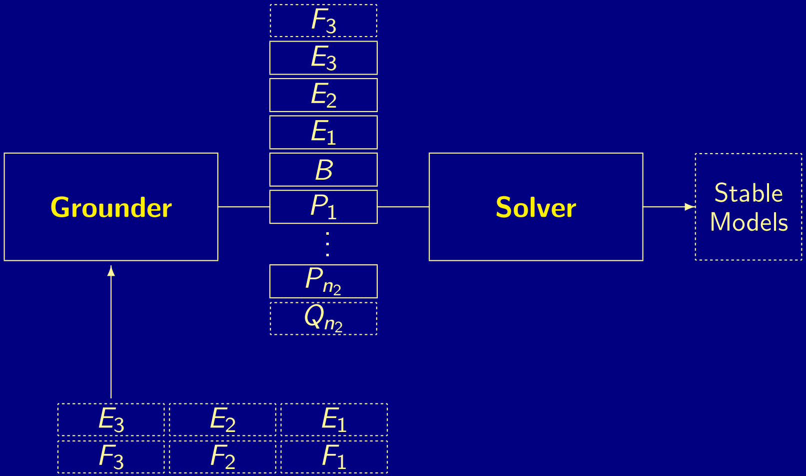
Reactive ASP Solving Process



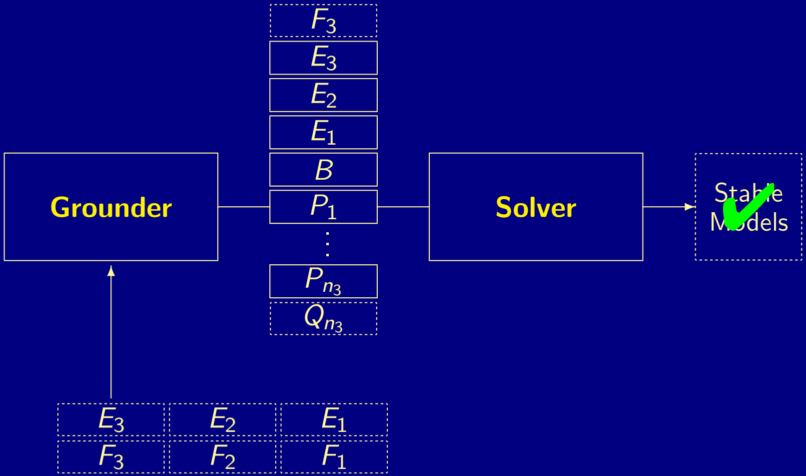
Reactive ASP Solving Process



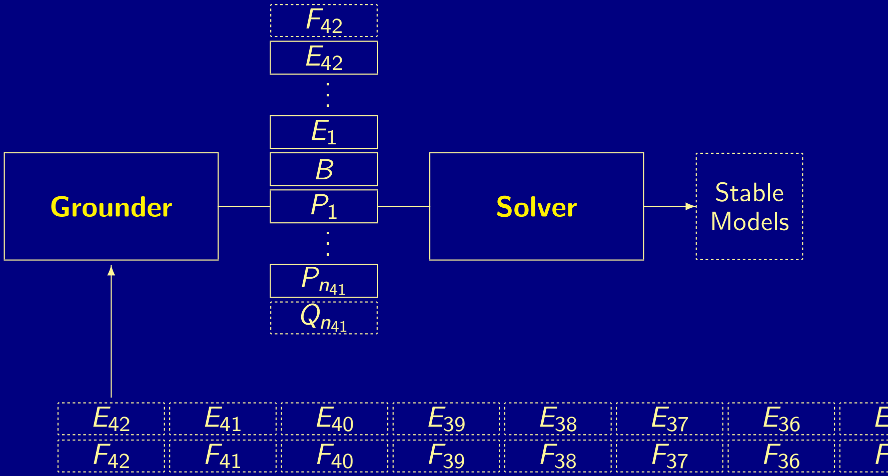
Reactive ASP Solving Process



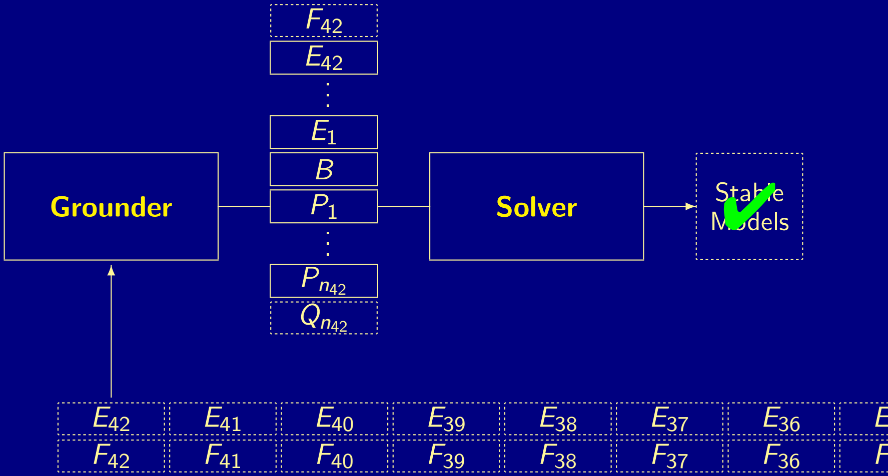
Reactive ASP Solving Process



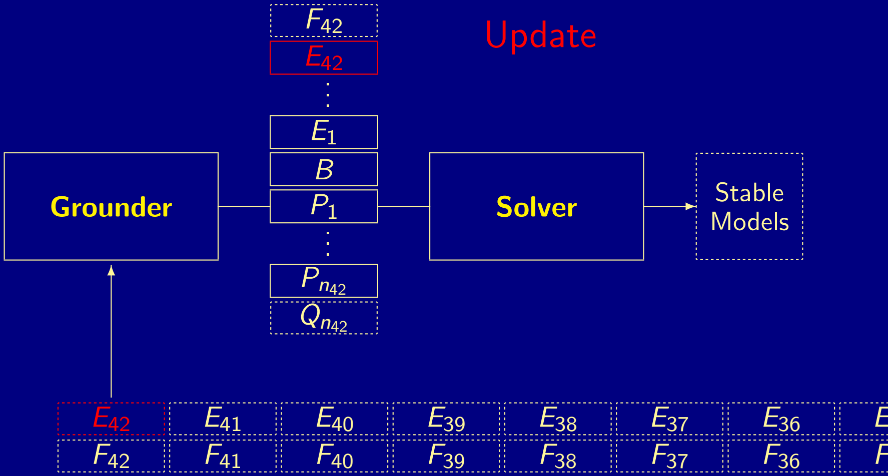
Reactive ASP Solving Process



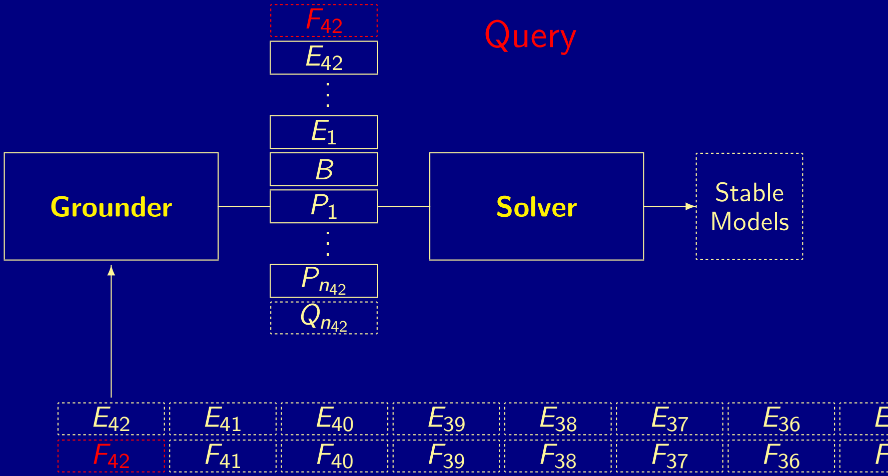
Reactive ASP Solving Process



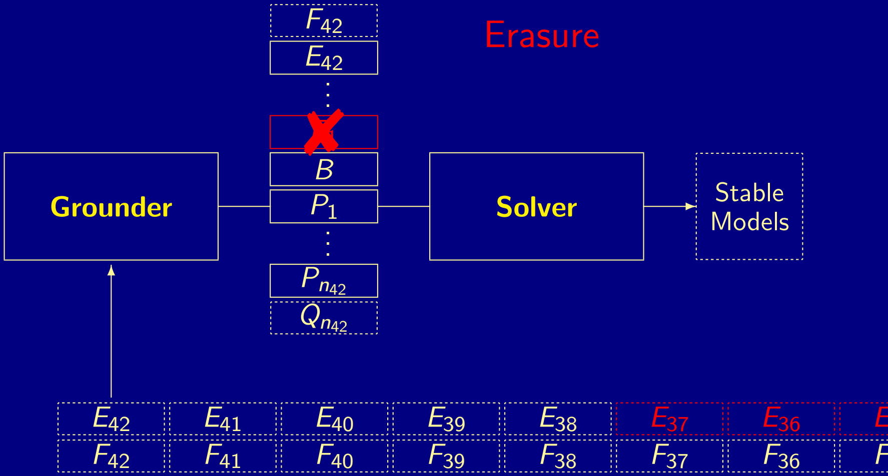
Reactive ASP Solving Process



Reactive ASP Solving Process



Reactive ASP Solving Process



Elevator Control

```
#base.
floor(1..3).
atFloor(1,0).

#cumulative t.
#external request(F,t) : floor(F).
1 { atFloor(F-1;F+1,t) } 1 :- atFloor(F,t-1), floor(F).
:- atFloor(F,t), not floor(F).
requested(F,t) :- request(F,t),     floor(F), not atFloor(F,t).
requested(F,t) :- requested(F,t-1), floor(F), not atFloor(F,t).
goal(t) :- not requested(F,t) : floor(F).

#volatile t.
:- not goal(t).
```

Pushing a button

- oClingo acts as a server listening on a port waiting for client requests
- To issue such requests, a separate controller program sends online progressions using network sockets
- For instance,

```
#step 1.
request(3,1).
#endstep.
```

- This process terminates when the client sends

```
#stop.
```

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  waiting for client requests
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